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DC モータ速度の適応制御系のオートチューニング

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Auto-Tuning for Adaptive Control System of DC Motor Speed

by

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Abstract

The auto-tuning method for the strongly stable adaptive control of DC motor speed system with great variation of the load inertia is proposed. The stable closed-loop characteristic polynomial that is designed by type-1 optimal servo with one sample delay is specified for the adaptive pole placement control. The appropriate adaptive control system can be derived, by adjusting automatically the weight of performance criterion in the optimal servo by means of fuzzy inference on the basis of the stability index. Furthermore, the transient characteristic is improved by tuning the tracking model according to certain relation between the performance weight, the settling time and the tracking model. In addition, the numerical simulations are used to prove that the proposed methods provide satisfied performance.

Keywords: *DC motor, auto-tuning, adaptive motion control, optimal control, recursive identification, stability index*

1. Introduction

The realization of an intelligent auto-tuning system on the motor control field is tried actively. The control performance usually deteriorates in the conventional motor speed control system with fixed controller when the load inertia changes greatly. It is well known that the adaptive control [1], [2] is very effective strategy for such systems. However, when the load inertia changes over a wide range, the compensator having unstable poles frequently appears in the adaptive pole placement control of DC motor speed, even if the closed-loop characteristic has been designed as a stable system. The appearance of unstable compensator is not desirable with respect to both stability and reliability. The unstable controller is used seldom, especially if the plant itself is a stable system. Therefore, the appropriate selection of a closed-loop pole is required in order to obtain a stable controller over a full range of the load inertia variation. However, selecting the closed-loop pole that ensures the stability of a compensator under the condition of the load inertia variation over a wide range is very difficult in the pole placement control. Therefore, if the designer is able to adjust the stable poles of a closed-loop system recursively according to the load inertia variation, the construction of a strongly stable system [3] is easily realized. Such research is an indispensable significant theme to the development of an intelligent auto-tuning technology.

Recently, authors announced some effective design methods [4]-[6], that construct a strongly stable adaptive pole placement system when the plant parameter greatly changes. It is main characteristic of the strategy that the proposed adaptive system evaluates the relative stability of both series compensator and optimal servo system by introducing a stability index. The procedure of the proposed method is summarized as follows. First, type-1 optimal servo [7] is recursively designed by estimating DC motor drive system. The pole placement controller is automatically constructed by solving *Diophantine* equation on the basis of the characteristic polynomial of the derived optimal servo. After the stability index of a series compensator is examined, the proposed method automatically adjusts the performance weight of an optimal servo by using the fuzzy inference [8] so that the adaptive system can place the stability index of the series compensator into the specified region. Furthermore, the desired performance of both transient response and manipulated variable can be achieved by tuning the tracking model in consideration of the relation between the performance weight in optimal control design and the settling time. Consequently, this strategy not only ensures the stability of both the closed- loop system and the controller but also can achieve a fine control performance.

2. Driver amplifier for DC motor

In this paper, DC motor is driven by introducing the servo amplifier based on type-1 optimal servo design as shown in Fig. 1, though the phase compensation design is popular. It is a reason of adoption that the optimal servo is most suited to the proposed method. The control of equivalent disturbance

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and the reduction of uncertainty are realized by using this servo driver, and the original performance of adaptive control is skillfully achieved. The symbols used in the design of DC motor driver are defined as follows:

L , R_a : armature inductance $[H]$ and resistance $[\Omega]$
i : armature current $[A]$, K_p : power amplifier gain
K_τ : torque constant $[Nm/A]$
K_e : back electromotive force constant $[V\sec/rad]$
J_m : momentum of rotor inertia $[kgm^2]$
J_ℓ : momentum of load inertia $[kgm^2]$
ω : rotor speed $[rad/\sec]$, τ_f : disturbance torque $[Nm]$
ω_V : voltage output of rotor speed $[V]$
S_V : conversion constant ω to ω_V $[V\sec/rad]$
r : applied voltage of power amplifier $[V]$
R_i : resistance for armature current detection $[\Omega]$

The state-space description of DC motor is expressed by disregarding a disturbance torque as follows:

$$\begin{cases} \dot{x} = A\,x + B\,r \\ y = C\,x \end{cases}, \quad x = [\omega \quad i\,]^T, \quad y = \omega_V \ , \tag{1}$$

where A, B and C are defined as

$$A = \begin{bmatrix} 0 & K_\tau/J \\ -K_e/L & -R_t/L \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ K_p/L \end{bmatrix}, \quad C = [S_V \quad 0],$$

$$J = J_m + J_\ell \ , \quad R_t = R_a + R_i \ . \tag{2}$$

Type-1 servo system is used for DC motor speed control. The extended system introduced with an integrator is described as

$$\begin{bmatrix} \dot{x} \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z_1 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r \,, \tag{3}$$

and actuating value is constructed by the following control:

$$r = -F\,x + K_1\,z_1 \ , \quad F = [f_1 \quad f_2] \,, \tag{4}$$

where f_1, f_2 and K_1 are feedback gains, respectively.

In general, the feedback gains can be decided by the pole placement method or the optimal servo design. The procedure of type-1 optimal servo design is summarized as follows. The extended deviation system of DC motor described as (1) is given by following equations:

$$\begin{cases} \dot{X} = A_1\,X + B_1\,v \\ e = C_1\,X \end{cases}, \quad X = \begin{bmatrix} x - x_S \\ r - r_S \end{bmatrix}, \quad e = y - u_S \,, \tag{5}$$

where the variables x_S, r_S and u_S represent the steady state of the variables x, r and u, respectively, and A_1, B_1 and C_1 are defined as

$$A_1 = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = [C \quad 0] \ . \tag{6}$$

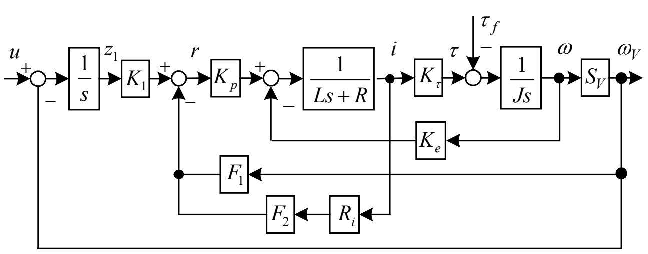

Fig. 1 The block diagram of DC motor driver

At this time, the optimal servo problem is considered an optimal regulator problem, which operates the following feedback control to the system of (5):

$$v = -F_X X \,, \quad F_X = [F \quad K_1] \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} . \tag{7}$$

Therefore, when the performance criterion is defined as

$$J = \int_0^\infty (X^T C_1{}^T W C_1{}^T X + v^T R_c\,v)dt \,, \quad W > 0, R_c > 0 \ , \tag{8}$$

the optimal feedback v^o that minimizes (8) is given by

$$v^o = -F_X{}^o X \,, \quad F_X{}^o = R_c{}^{-1} B_1{}^T P_c^o \,, \tag{9}$$

where P_c^o is the solution of following *Riccati* equation:

$$A_1{}^T P_c + P_c A_1 - P_c B_1 R_c{}^{-1} B_1{}^T P_c + C_1{}^T W C_1 = 0 \,. \tag{10}$$

After the feedback gains in (9) were obtained, the practical feedback is performed by using the following optimal gain:

$$[F \quad K_1] = F_X{}^o \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}, \quad F_1 = f_1/S_V \ , \quad F_2 = f_2/R_i \,. \tag{11}$$

Furthermore, the closed-loop system is described as

$$\begin{bmatrix} \dot{x} \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} A - BF & BK_1 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \ , \quad \omega_V = C\,x \,. \tag{12}$$

Consequently, the transfer function to the motor speed ω_V from the reference u can be obtained as follow:

$$\frac{\omega_V}{u} = \frac{K}{JLs^3 + J(R + K_p f_2)s^2 + K_\tau(K_p f_1 + K_e)s + K} \ , \tag{13}$$

where the constant gain K is $K = K_1 K_p K_\tau S_V$.

3. Adaptive control system

3.1 Pole placement control system

The proposed adaptive control system shown in Fig. 2 does not introduce the pole-zeros cancellation because it is really difficult to avoid the unstable zeros in the discrete modeling of the practical plant. It is also constructed with the integrator in order to reject a stationary disturbance.

DC motor system is described by the ARX model:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + w(k) \ , \tag{14}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \ , \tag{15}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \ , \tag{16}$$

where $w(k)$ is the white noise having zero mean. The reference signal is given by the tracking model:

$$A_m(q^{-1})r_m(k) = q^{-d}B_m(q^{-1})u_m(k) \tag{17}$$

$$A_m(q^{-1}) = 1 + a_{m1} q^{-1} + \cdots + a_{m\ell} q^{-\ell} \ , \tag{18}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1} q^{-1} + \cdots + b_{m\ell} q^{-\ell} \ . \tag{19}$$

At this time, the regulation performance of a stable closed-loop system is specified by the following:

$$D_o(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_{nd} q^{-nd} \,, \quad nd \le n + m + d \ . \tag{20}$$

In addition, the tracking performance of the system is achieved by using prefilter $T(q^{-1})$ such as

$$T(q^{-1}) = D_o(1)/B(1) \tag{21}$$

or the polynomial such as

$$T(q^{-1}) = D_o(q^{-1})/B(1) \ . \tag{22}$$

Here, let us consider the difference described by

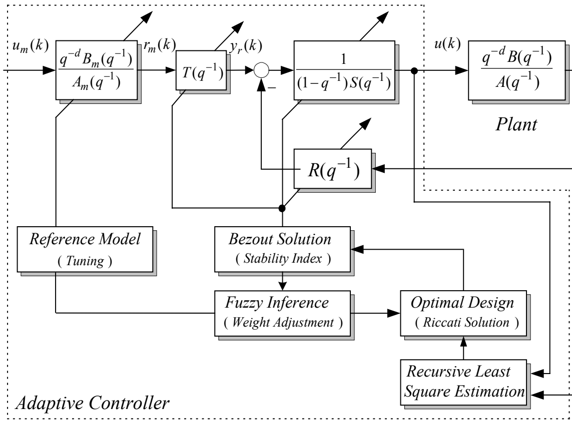

Fig. 2 Block diagram of adaptive control system

$$e(k) = D_o(q^{-1})y(k) - T(q^{-1})B(q^{-1})r_m(k) \ . \tag{23}$$

The variance of (23) is the performance criterion:

$$J = E\,[e^2(k)] \ . \tag{24}$$

Then, the optimal input signal $u(k)$ that minimizes J is constructed by means of the following value:

$$u(k) = \frac{T(q^{-1})r_m(k+d) - R(q^{-1})y(k)}{(1-q^{-1})S(q^{-1})} \ , \tag{25}$$

where $S(q^{-1})$ and $R(q^{-1})$ are given by the polynomials:

$$S(q^{-1}) = 1 + s_1 q^{-1} + \cdots + s_{ns} q^{-ns} \ , \quad ns = m + d - 1 \ , \tag{26}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \cdots + r_{nr} q^{-nr} \ , \quad nr = n \ . \tag{27}$$

Furthermore, both controller $S(q^{-1})$ and $R(q^{-1})$ can be derived by solving *Diophantine* eqation:

$$D_o(q^{-1}) = (1-q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) \ . \tag{28}$$

3.2 Recursive identification system

When the parameter of DC motor model (14) is unknown or greatly varied, the estimated value of a plant parameter is substituted in the control system design. If the parameter vector θ and the regression vector $\varphi(k)$ constructed by measurement data are defined as

$$\theta^T = \{a_1, a_2, \cdots, a_n, b_0, b_1, \cdots, b_m\} \tag{29}$$

and

$$\varphi^T(k) = \{-y(k-1), -y(k-2), \cdots, -y(k-n), \\ u(k-d), u(k-1-d), \cdots, u(k-m-d)\} \ , \tag{30}$$

respectively, then, the plant output is expressed by

$$y(k) = \varphi^T(k)\theta + v(k) \ . \tag{31}$$

Here, let us use the notation $\hat{y}(k\,|\,\theta)$ as the one- step-ahead prediction value, then it is given by the linear formula with respect to a parameter vector θ as follows:

$$\hat{y}(k\,|\,\theta) = [1 - A(q^{-1})]y(k) + B(q^{-1})u(k) = \varphi^T(k)\theta \ . \tag{32}$$

The estimate model output is constructed according to

$$y_m(k) = \varphi^T(k)\hat{\theta}(k) \ , \tag{33}$$

where the estimate vector of the parameter is defined by

$$\hat{\theta}^T(k) = \{a_1(k), a_2(k), \cdots, a_n(k), \\ b_0(k), b_1(k), \cdots, b_m(k)\} \ . \tag{34}$$

If the condition of signal to noise (S/N) ratio of the plant described by ARX model is good, the least-squares estimation is reliable and has few biases with respect to the estimated value. Consequently, the recursive identification based on least-squares method (RLS) is described by means of the following formulation:

RLS algorithm

parameter adjusting:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{1 + \varphi^T(k)P(k-1)\varphi(k)}\varepsilon(k) \ , \tag{35}$$

adaptive gain:

$$P(k) = \frac{1}{\lambda_1(k)}\left\{P(k-1) - \frac{\lambda_2(k)P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda_1(k) + \lambda_2(k)\varphi^T(k)P(k-1)\varphi(k)}\right\} , \tag{36}$$

apriori error:

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \ , \tag{37}$$

where weighting sequences $\lambda_1(k)$ and $\lambda_2(k)$ in (36) are $0 < \lambda_1(k) \le 1$ and $0 \le \lambda_2(k) < 2$, respectively. The designer can obtain other adaptive gain that has the typical characteristic by selecting the appropriate values for $\lambda_1(k)$ and $\lambda_2(k)$.

3.3 Characteristic polynomial based on optimal servo

DC motor estimated system of (14) is transformed to the state-space description $\Sigma : (\overline{A}, \overline{B}, \overline{C})$ ignoring $w(k)$. Here, a quadratic type performance criterion is defined as

$$J = \sum_{k=0}^{\infty}\{\tilde{x}^T(k)\mathrm{Q}\tilde{x}(k) + R\tilde{v}^2(k)\} \ , \qquad R > 0 \ , \tag{38}$$

where $\tilde{x}(k)$ and $\tilde{v}(k)$ represent the states and actuating value for the extended deviation system, respectively, and Q is a semi-positive definite matrix. Type-1 optimal servo having one sample controller delay that minimizes a performance index of (38) is given as follows:

***Riccati* equation:**

$$P = \mathrm{Q} + \overline{A}^T P\overline{A} - \overline{A}^T P\overline{B}(R + \overline{B}^T P\overline{B})^{-1}\overline{B}^T P\overline{A} \ , \tag{39}$$

where Q and R represent the weights of the quadratic type performance criterion, respectively.

Optimal feed-back gain:

$$F = (R + \overline{B}^T P\overline{B})^{-1}\overline{B}^T P\overline{A} \ , \tag{40}$$

Controller parameters:

$$g = F\overline{B} + 1 \tag{41}$$

$$[H, K] = [F\overline{A}^2, F\overline{A}\overline{B} + F\overline{B} + 1]E^{-1} \ , \tag{42}$$

$$E = \begin{bmatrix} \overline{A} - I & \overline{B} \\ \overline{C} & 0 \end{bmatrix} \ , \tag{43}$$

Furthermore, the state-space description of the optimal servo system is expressed by

$$\begin{bmatrix} x(k+1) \\ u(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} & 0 \\ -H & -g & K \\ -\overline{C} & 0 & 1 \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k) \ . \tag{44}$$

Therefore, the characteristic polynomial $D_0(q^{-1})$ of closed-loop shown in Fig. 3 is, for example, calculated by *Faddeev's* algorithm from the system matrix of (44).

3.4 Auto-Tuning based on the stability index

The tuning method for the strongly stable pole placement based on the stability index is discussed. The stability index is introduced in order to evaluate the relative stability of the control system. In the following n^{th} -order characteristic polynomial of the continuous-time transfer function:

$$p(s) = f_n s^n + \cdots + f_1 s + f_0 \ , \quad n \geq 2 \ , \tag{45}$$

the stability indices γ_i are generally defined as follows:

$$\gamma_i = f_i^2 \big/ f_{i+1} \cdot f_{i-1} \ , \quad (i = 1, \cdots, n-1) \ . \tag{46}$$

A series compensator $[S(q^{-1})]^{-1}$ is derived by solving (28), according to the characteristic polynomial $D_o(q^{-1})$ that is obtained with an optimal design of the previous section.

Next, in order to apply the stability indices γ_i to the discrete-time series compensator $[S(q^{-1})]^{-1}$, it is transformed into the continuous-time transfer function $S_c(s)$ by introducing the inverse bilinear transformation

$$q = (2 + Ts) \big/ (2 - Ts) \ , \tag{47}$$

where T is the sampling period.

After this operation, the stability indices γ_i in terms of the denominator of continuous-time compensator $S_c(s)$ are calculated. Furthermore, the simplest index γ is selected by means of the algebraic product (48) of stability indices γ_i in order to evaluate the relation between the stability of controller and the performance of closed-loop system.

$$\gamma = \prod_{i=1}^{n-1} \gamma_i \ . \tag{48}$$

This new index γ can be related to the performance weight R in the optimal servo. Furthermore, the index γ is related to each characteristic, whether open-loop, such as gain-phase margin, or close-loop, such as settling time. Appropriate auto-tuning of weight R based on the resultant stability index γ is achieved, by considering the relations as mentioned above. Namely, the fuzzy inference is introduced to adjust the performance weight R and this stability index γ is effectively used as the scaling factor.

In addition, in order to complete the defuzzification of the inference result, the consequent of fuzzy inference is executed using the gravity method of *Mamdani* as follows:

$$u_m(k-1) = \{\Sigma x_j \cdot \mu(x_j)\} \big/ \Sigma \mu(x_j) \ , \tag{49}$$

where x_j is the nonfuzzy value, $\mu(x_j)$ is the value of the membership function, and $u_m(k-1)$ is the inference result. In order to place the stability index γ of the series compensator $S_c(s)$ into the specified region, the weight R of a performance criterion in the optimal servo system is tuned by means of the following recursive formula:

$$R(k) = R(k-1) \cdot 10^{\beta(k-1)} \ , \tag{50}$$

$$\beta(k-1) = c \cdot u_m(k-1) \ , \quad c\text{:}\ any\ constant \ . \tag{51}$$

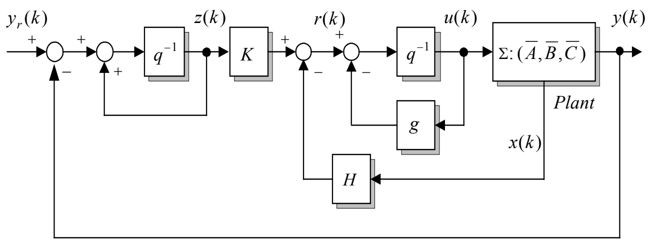

Fig. 3 Type-1 optimal servo with one sample delay

4. DC Motor speed control system

The major constants of DC motor are shown in Table 1. The servo driver amplifier of DC motor shown in Fig.1 is constructed by using an optimal servo design under the load inertia $J = 1.5 J_m$ ($J_\ell = 0.2 \times 10^{-4}$). When both performance weights W and R of (8) are selected as $W = 60$ and $R = 1/150$, respectively, the optimal feedback gains are obtained as $F_1 = 1.19$, $F_2 = 0.276$ and $K_1 = 94.9$.

The proposed method is verified by using the reduced order discrete-time model, because DC motor speed system described by (13) is possible to reduce the system order by examining the singular value. The new method is easily designed by using both discrete-time models of plant $\Sigma_f (J_f \cong 1.5 J_m)$ and $\Sigma_s (J_s \cong 5 J_f)$ in order to adapt to great variation of the load inertia. When the sampling time T is selected as $0.005[\sec]$, the discrete-time models of Σ_f and Σ_s are estimated by the recursive least-squares method in section 3.2, respectively, as

$$\Sigma_f : \ G_f(q^{-1}) = \frac{q^{-1}(0.028214 + 0.17243 q^{-1})}{1 - 1.2920 q^{-1} + 0.49368 q^{-2}} \ , \tag{52}$$

$$\Sigma_s : \ G_s(q^{-1}) = \frac{q^{-1}(0.012888 + 0.037583 q^{-1})}{1 - 1.8154 q^{-1} + 0.86604 q^{-2}} \ . \tag{53}$$

Due to the computation time delay, d should be increased by 1, yielding $d = 2$. The closed-loop system of second order is derived by optimal servo design in the case of these motor design models. It has been confirmed by checking stability index that the sufficient stability could not be ensured in the case of this control characteristic. Therefore, appending the virtual pole-zero pair $P_V = 0.8$ to the plant $G_f(q^{-1})$, in order to design the closed-loop polynomial $D_o(q^{-1})$ of the third order, type-1 optimal servo is calculated under a condition that the weights are $Q = diag\ (100,100,100)$ and $R = 5 \times 10^4$, respectively.

Then, the controller parameters are obtained, respectively, as

$$g = 1.17 \ , \quad K = 1.99 \ , \quad H = [0.1863 \ \ -1.365 \ \ 1.326] \ . \tag{54}$$

The characteristic polynomial is calculated as

$$D_o(q^{-1}) = 1 - 1.9245 q^{-1} + 1.3355 q^{-2} - 0.3310 q^{-3} \ . \tag{55}$$

$S(q^{-1})$ and $R(q^{-1})$ are given, respectively, as

$$S(q^{-1}) = 1 + 0.36752 q^{-1} + 0.33525 q^{-2} \tag{56}$$

$$R(q^{-1}) = 2.0153 - 2.5767 q^{-1} + 0.95985 q^{-2} \ . \tag{57}$$

Table 1 The design constants of DC motor

DC motor 24V-20W	
$K_\tau = 7.154 \times 10^{-2} [Nm/A]$	$K_e = 7.162 \times 10^{-2} [V \sec/rad]$
$J_m = 0.4 \times 10^{-4} [kgm^2]$	$S_v = 3.183 \times 10^{-2} [V / rad / \sec]$
$R_a = 4.3[\Omega]$, $R_i = 0.2[\Omega]$	$K_P = 15$, $L = 6$ $[mH]$

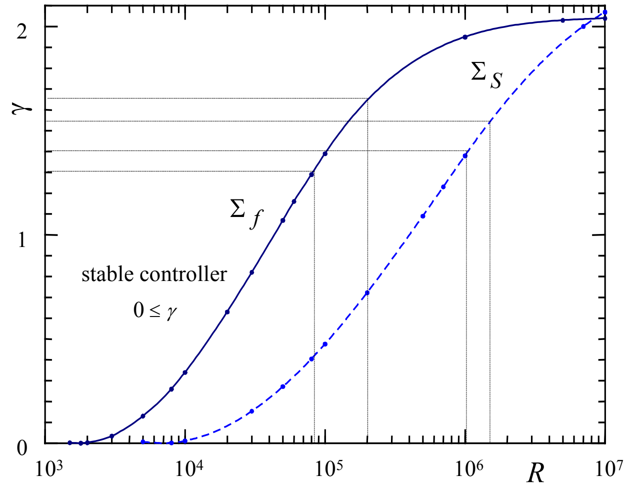

Fig. 4 Each index γ of plants Σ_f and Σ_s to weight R

Furthermore, the stability index is obtained as $\gamma = 1.073$, and also in order to confirm the absolute stability of $F_c(s)$, *Hurwitz* determinant is calculated as $H_1 = 532$. The gain margin g_m and the phase margin p_m are calculated from the loop transfer function of this discrete-time system as

$$g_m = 8.99\,[dB]\ ,\qquad p_m = 67.2\,[\text{deg}], \tag{58}$$

respectively. In addition, settling time is confirmed as ST =12 [sample] through a step response simulation of only closed-loop system. Similarly, when the calculation is repeated changing R continuously, using fixed Q in terms of both weights, Q and R, each significant characteristic is obtained. The index γ and the settling time ST to the weight R of both plants, Σ_f and Σ_s, are shown, respectively, in Figs. 4 and 5. Next, the relation of both weight R and stability index γ having good performance for the control system of Σ_f and Σ_s is examined by considering each characteristic. Consequently, one of the appropriate ranges for the auto-tuning is selected as

$$\left.\begin{array}{lll} l_f \le \gamma_f \le u_f, & l_f = 1.3, & u_f = 1.65 \\ l_s \le \gamma_s \le u_s, & l_s = 1.4, & u_s = 1.55 \end{array}\right\}. \tag{59}$$

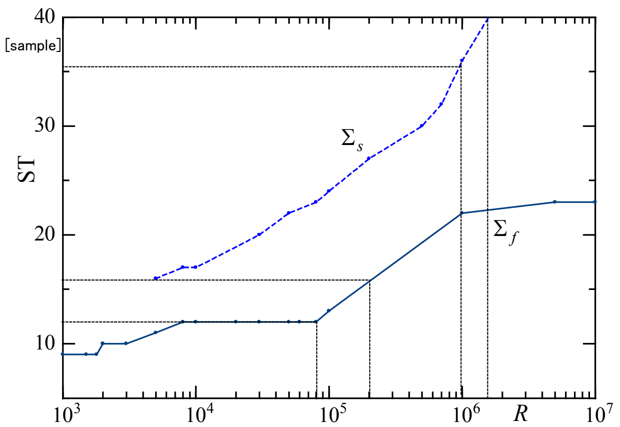

Fig. 5 Each settling time of plants Σ_f and Σ_s to weight R

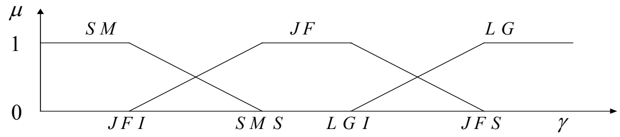

Fig. 6 Membership function of antecedent

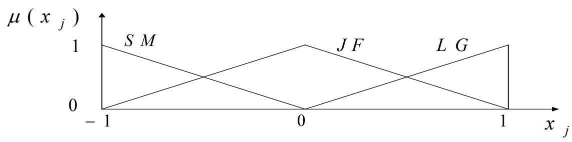

Fig. 7 Membership function of consequent

Fuzzy inference is used to ensure the stability index γ of compensator $[S(q^{-1})]^{-1}$ within a specified area.

The fuzzy variable γ used in the antecedent of the fuzzy rule has a trapezoidal membership function, as shown in Fig. 6. The membership function used in the consequent of the fuzzy rule has a triangular form, as shown in Fig. 7. The horizontal scale in the membership function of Fig. 6 can be selected as

$$\begin{array}{ll} JFI = \min(\gamma_f \cup \gamma_s)\ , & SMS = \min(\gamma_f \cap \gamma_s) \\ LGI = \max(\gamma_f \cap \gamma_s)\ , & JFS = \max(\gamma_f \cup \gamma_s)\ . \end{array} \tag{60}$$

The complete inference is found by calculating (37) in terms of membership function of Fig. 7.

The transient characteristic is improved by

$$A_m(q^{-1}) = (1 - \alpha\, q^{-1})^3\ ,\quad B_m(q^{-1}) = A_m(1)\ , \tag{61}$$

where α is the adjustable parameter of $\alpha < 1$. When the prefilter of (22) is specified, the adaptive system has the tracking specification:

$$G_{tr} = \frac{q^{-2} A_m(1)}{A_m(q^{-1})} \cdot \frac{B(q^{-1})}{B(1)}\ . \tag{62}$$

By considering the characteristic of (62), it is confirmed that both plants Σ_f and Σ_s have similar settling time, even if the load inertia changes greatly. Therefore, the transient response can be improved by tuning the pole α of tracking model accordingly. The tuning formula is given by

$$\alpha(k) = a \log R(k) + b\ ,\quad a = 0.259\ ,\quad b = -0.824\ . \tag{63}$$

5. Simulation results

Two simulation results for the adaptive control of DC motor speed are shown. As the simulation condition, the performance weight Q is chosen to be the fixed value of $Q = diag\ (100,100,100)$ and the initial value of R is also chosen to be $R = 1\times10^6$. The load inertia of DC motor has been changed to Σ_s from Σ_f at a step of 400 and a disturbance noise of variance 5×10^{-6} has been added to the simulation. First, the simulation results without the tracking model of (17) are shown in Figs. 8 through 10. The performance weight R is adjusted appropriately and the required controller can automatically be obtained in each load inertia even after the load inertia are changed greatly at a step of 400 in Fig. 9. Furthermore, Fig. 10 reveals that the proposed adaptive control system is appropriately adjusted by means of fuzzy inference as the stability index γ is put in the specified region. However, an overshoot is observed in the transient response in Fig. 8 a little.

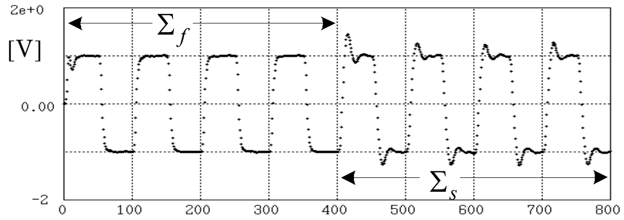

Fig. 8 Controlled variable without tracking model

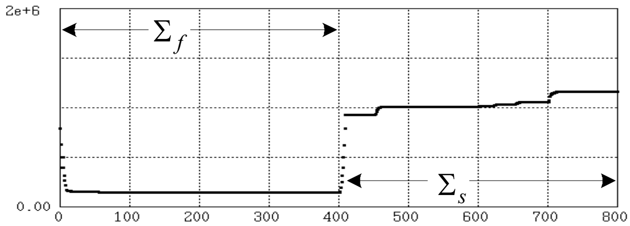

Fig. 9 Weight R without tracking model

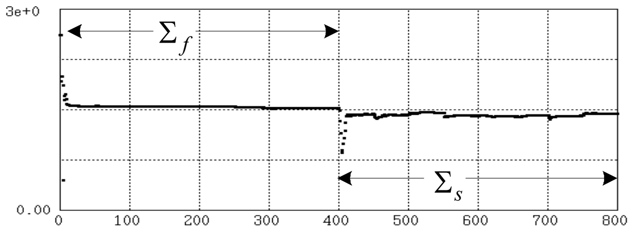

Fig. 10 Stability index γ without tracking model

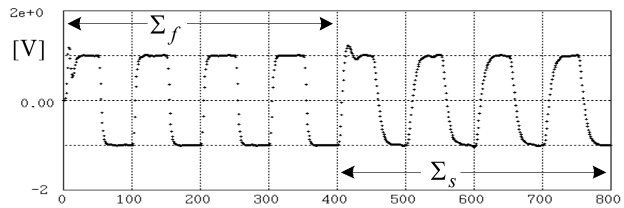

Fig. 11 Controlled variable with tracking model

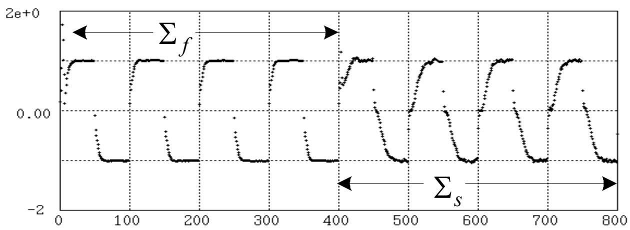

Fig. 12 Actuating value with tracking model

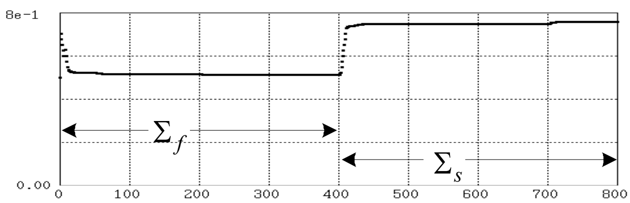

Fig. 13 Tracking model pole α by proposed system

When the prefilter of (22) is specified and the tracking model of (17) is used, the transient response is finally improved as shown in Figs. 11 through 13. The controlled variable shows a good transient response in Fig. 11 and also the overshoot disappears after the completion of parameter identification, simultaneously an actuating value is suitably given without the disturbance as shown in Fig.12. Furthermore, the stable pole α is appropriately tuned as shown in Fig.13. The performance weight and the index are almost similar to Figs. 9 and 10 in this simulation.

6. Conclusions

This paper has proposed a useful strategy for achieving the adaptive control of DC motor speed system with great variation of load inertia. The difficulty of conventional adaptive pole placement control is how to place the stable pole in the *Diophantine* equation in order to design the appropriate controller according to the load inertia of DC motor that widely changes. The proposed methods can place the stability index in the specified area and then, overcome the problem of unstable series compensator that appears in conventional adaptive control system. The evident solution for the control purpose is easily achieved by applying the proposed scheme. Both operations of the estimation and the control were recursively repeated in order to inspect the real time performance. We append to the conclusion that the proposed method is sufficiently applicable to practical DC motor system.

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