SELF-TUNING CONTROL FOR ROTATIONAL INVERTED PENDULUM BY EIGENVALUE APPROACH

Phornsuk RATIROCH-ANANT [*], Masatoshi ANABUKI [**] and Hiroshi HIRATA [**]

* Faculty of Engineering, King Mongkut's Institute of Technology, Ladkrabang, THAILAND

** School of Information Technology and Electronics, Tokai University, JAPAN

ABSTRACT

The self-tuning strategy concerning the stabilized control system of the rotational inverted pendulum is considered. In the case of the pendulum with expansion and contraction under the stabilization, the system parameter is recursively estimated and the redesigned controller is appropriately updated. The available disturbance torque is appended to the control signal because the inverted pendulum does not have sufficient information for the estimation of the system parameter under the stabilization. It is verified by the practical experiment that the proposed self-tuning is very useful as one of the on-line tuning.

1. INTRODUCTION

So many papers concerning the stabilization of the inverted pendulum are reported. However, few methods consider the case when the pendulum expands and contracts under the stabilization. The on-line estimation for a little input and output information system is the difficult problem and important theme in the auto-tuning.

This paper considers the self-tuning control method [1] for the rotational inverted pendulum that the momentum of inertia of the pendulum part is widely changed. The controller is first designed by the optimal regulator using the system parameters estimated during the downward movement of the pendulum. After the controller stabilizes the inverted pendulum and its length is also changed, the little information of the stabilized system could not achieve the system parameter estimation according to the changing length. By the recursive least square (RLS) method under the condition that the available disturbance torques are shortly appended to the control signal, the stabilized system parameters can be estimated. The motion data by this approach, for example, the joint angles, the angular velocities and the angular accelerations are necessary in order to estimate the pendulum system. However, only the motor current and the joint angles are used as the measurement data, other motion data are obtained by using the derivative [2],[3] low pass filter.

When the identification system has different sensor dynamics, the estimation accuracy generally deteriorates. Therefore, the useful filter structure [3],[4] for compensating the difference of the sensor characteristics is introduced. Furthermore, the validity of the convergence concerning the estimated parameter in the proposed self-tuning is verified by calculating the eigenvalues [5] of a pendulum system.

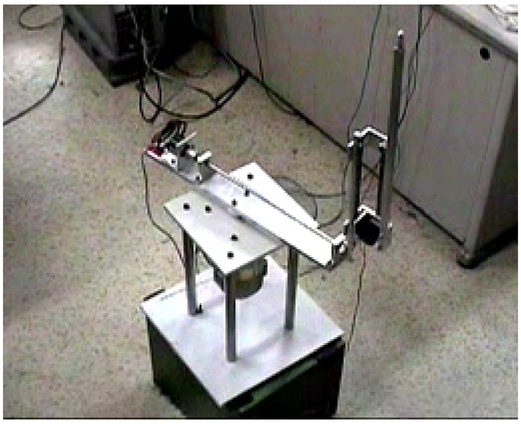

Figure 1 Rotational inverted pendulum system.

2. DYNAMICS OF INVERTED PENDULUM

The rotational inverted pendulum system used in the experiment and its schematic drawings are shown in Figure 1 and Figure 2, respectively. The rotational arm is directly driven by D.C. motor (100V-250W) and also the link of pendulum is driven to straight by rack and pinion gear connected to small D.C. motor (24V-6.4W). Dynamic equation of such rotational inverted pendulum can be written in the following general form

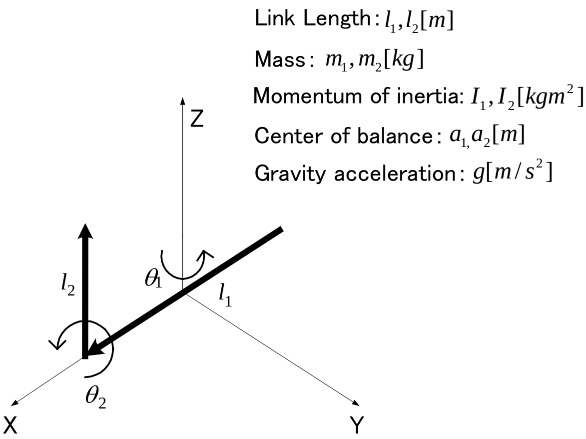

Figure 2 Schematic drawing of inverted pendulum.

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$$\tau = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) + B\dot{\theta} + D(\dot{\theta}) \ , \qquad (1)$$

where τ is the motor torques, and θ, $\dot{\theta}$ and $\ddot{\theta}$ are the joint angles, velocities and accelerations, respectively. $M(\theta)$ is the inertia matrix, $C(\theta,\dot{\theta})\dot{\theta}$ represents the torques arising from centrifugal and Coriolis forces. $B\dot{\theta}$, $D(\dot{\theta})$ and $G(\theta)$ represent the friction torques acting at rotational joints, and the gravitational torques respectively. In the case of neglecting the Coulomb friction of the pendulum part, the terms in (1) can be obtained by means of the Euler- Lagrange dynamic equation as follows:

$$M(\theta) = \begin{bmatrix} J_1 + J_2 S_2^{\,2} & -rl_1 C_2/2 \\ -rl_1 C_2/2 & J_2 \end{bmatrix} \ , \qquad (2)$$

where $J_1 = I_1 + m_1 a_1^{\,2} + m_2 l_1^{\,2}$, $J_2 = I_2 + m_2 a_2^{\,2}$, $r = m_2 a_2$,
$S_2 = \sin(\theta_2)$, $C_2 = \cos(\theta_2)$, $\theta = [\theta_1 \quad \theta_2]^T$
and

$$C(\theta,\dot{\theta})\dot{\theta} = \begin{bmatrix} 2J_2 S_2 C_2 \dot{\theta}_1 \dot{\theta}_2 + rl_1 S_2 \dot{\theta}_2^{\,2}/2 \\ -J_2 S_2 C_2 \dot{\theta}_1^{\,2} \end{bmatrix},$$

$$G(\theta) = \begin{bmatrix} 0 \\ -rgS_2 \end{bmatrix}, \ B\dot{\theta} = \begin{bmatrix} b_1\dot{\theta}_1 \\ b_2\dot{\theta}_2 \end{bmatrix}, \ D(\dot{\theta}) = \begin{bmatrix} d_1 \,\mathrm{sgn}(\dot{\theta}_1) \\ 0 \end{bmatrix} . \qquad (3)$$

Furthermore, the dynamic equation (1) can be described by the linear parameterize form

$$\tau = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) + B\dot{\theta} + D(\dot{\theta}) = \boldsymbol{\Phi}^T(\theta,\dot{\theta},\ddot{\theta})\delta \ , \qquad (4)$$

where the joint torque is $\tau = [\tau_1 \ \ 0]^T$ and the regressor $\Phi(\theta,\dot{\theta},\ddot{\theta})$ and the basic parameter δ are given by

$$\boldsymbol{\Phi}^T(\theta,\dot{\theta},\ddot{\theta}) := \begin{bmatrix} \ddot{\theta}_1 & \phi_{12} & \phi_{13} & \dot{\theta}_1 & 0 & \mathrm{sgn}(\dot{\theta}_1) \\ 0 & \phi_{22} & \phi_{23} & 0 & \dot{\theta}_2 & 0 \end{bmatrix} , \qquad (5)$$

$$\phi_{12} = S_2^{\,2}\ddot{\theta}_1 + 2S_2C_2\dot{\theta}_1\dot{\theta}_2 \ , \quad \phi_{13} = l_1(S_2\dot{\theta}_2^{\,2} - C_2\ddot{\theta}_2)/2 \ ,$$
$$\phi_{22} = \ddot{\theta}_2 - S_2C_2\dot{\theta}_1^{\,2} \ , \quad \phi_{23} = -l_1C_2\ddot{\theta}_1/2 - gS_2$$

and

$$\delta := [J_1 \quad J_2 \quad r \quad b_1 \quad b_2 \quad d_1]^T \ . \qquad (6)$$

The joint torque is calculated by $\tau_1 = k_\tau \, \boldsymbol{i}$ using the torque constant k_τ and the motor current i. In the case with uncertainty in the torque constant, (4) is changed to (7) because it is useful for the estimation of the basic parameter included the torque constant.

$$\boldsymbol{i} = \boldsymbol{\Phi}^T(\theta,\dot{\theta},\ddot{\theta})\boldsymbol{\sigma} \ , \quad \boldsymbol{\sigma} = \boldsymbol{\delta} \ / k_\tau = [\sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6]^T \ . \ (7)$$

3. STATE EQUATION BY BASIC PARAMETER

The linear approximate equation of (1) in order to apply the optimal regulator design is given by

$$\begin{bmatrix} J_1 & -rl_1/2 \\ -rl_1/2 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -rg\theta_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} . \qquad (8)$$

Therefore, in the case of using such symbolic definition as the input $u = i$, the output $\boldsymbol{y} = [\theta_1 \ \ \theta_2]^T$ and the state variable $\boldsymbol{x} = [\theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2]^T$, the state space expression $\sum_S : (A, B, C)$ is described according to the following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{\sigma_2\sigma_4}{\det} & \dfrac{l_1 g \sigma_3^{\,2}}{2\det} & -\dfrac{l_1\sigma_3\sigma_5}{2\det} \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{l_1\sigma_3\sigma_4}{2\det} & \dfrac{g\sigma_1\sigma_3}{\det} & -\dfrac{\sigma_1\sigma_5}{\det} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \dfrac{\sigma_2}{\det} \\ 0 \\ \dfrac{l_1\sigma_3}{2\det} \end{bmatrix} \qquad (9)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad \det = \sigma_1\sigma_2 - \sigma_3^{\,2} l_1^{\,2}/4 \ . \qquad (10)$$

4. SELF-TUNING CONTROL SYSTEM

4.1 Optimal control system

The self-tuning control system of the inverted pendulum is shown by Figure 3. The feedback gain F is designed by the optimal regulator using the basic parameter obtained in the estimation part. Riccati equation is quickly solved by the eigenvector method using Hamilton matrix.

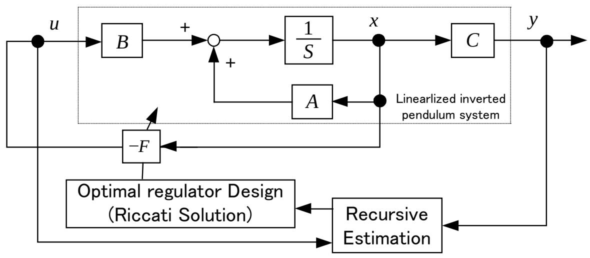

Figure 3 Optimal control system with tuning part.

4.2 Acquisition of regressor for estimation system

The effective filter structure shown in Figure 4 is introduced to compensate the difference of the sensor characteristics. The necessary motion data for the estimation of the pendulum inverted system is obtained by the following various filters [3],[4] .

$$\begin{cases} i_a = G_0 i \ , \quad \theta_a = G_0\theta \\ \mathrm{sgn}(\dot{\theta}_a) = G_0 \,\mathrm{sgn}(\dot{\theta}) \ , \ \dot{\theta} \cong \{\theta(k) - \theta(k-1)\}/T \\ \dot{\theta}_a = G_1\theta \ , \quad \ddot{\theta}_a = G_2\theta \end{cases} \qquad (11)$$

$$G_0 = \frac{1}{(1+\tau_f s)^3} \ , \quad G_1 = sG_0 \ , \quad G_2 = s^2 G_0 \qquad (12)$$

The operation of these filters is also calculated in discrete form using the bilinear transformation:

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \ , \quad T : sampling \ period. \qquad (13)$$

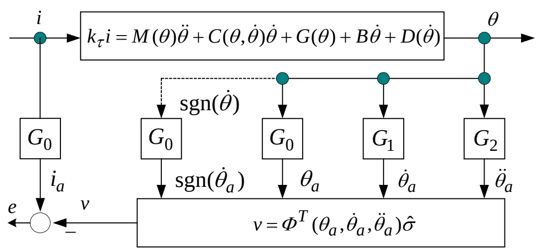

Figure 4 Regressor acquisition system for estimation.

4.3 Recursive estimation system

The estimate model is defined as

$$v(k) = \boldsymbol{\Phi}^T(k)\hat{\sigma}(k) \ , \qquad (14)$$

where $\hat{\sigma}(k)$ and $v(k)$ are, respectively, the estimated basic parameter and the output of the estimate model, $\boldsymbol{\Phi}(k)$ is the regressor of (5). The recursive estimation is completed in every sampling interval by filling the motion data

$$\{\theta_\ell(k),\ \dot{\theta}_\ell(k),\ \ddot{\theta}_\ell(k),\ i(k)\}\ ,\quad (\ell = 1,2)\,. \qquad (15)$$

When the condition in signal to noise ratio of the dynamical system is good, the following least-squares estimation has small bias concerning the estimation.

$$\hat{\sigma}(k) = \hat{\sigma}(k-1) - \frac{P(k)\boldsymbol{\Phi}(k)e(k)}{\lambda(k) + tr[\boldsymbol{\Phi}^T(k)P(k)\boldsymbol{\Phi}(k)]} \qquad (16)$$

$$P(k) = \frac{1}{\lambda(k)}\left\{P(k-1) - \frac{P(k-1)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)P(k-1)}{\lambda(k) + tr[\boldsymbol{\Phi}^T(k)P(k-1)\boldsymbol{\Phi}(k)]}\right\} \qquad (17)$$

$$e(k) = i(k) - \boldsymbol{\Phi}^T(k)\hat{\sigma}(k-1) \qquad (18)$$

$$\lambda(k) = (1-\mu)\lambda(k) + \mu \qquad (19)$$

where weighting sequences $\lambda(k)$ is usually chosen within $0.98 < \lambda(k) \le 1$ and μ is the constant for adjusting $\lambda(k)$.

5. VERIFICATION OF ESTIMATE ACCURACY

The stabilized system has little information for the estimation of the system parameter. Therefore, RLS estimation for the self-tuning system is verified by the numerical simulation under the condition that the available disturbance torques (20) is appended to the control signal after the upright of the pendulum.

$$u_i = 2\{\sin(\pi t) + \sin(2\pi t) + \sin(4\pi t) + \sin(8\pi t)\}\ \ [\text{A}] \qquad (20)$$

Table1 and Table2 are, respectively, the basic parameters of the inverted pendulum model and the condition used in the simulation of the recursive estimation. Table3 is also a part of the estimation result that shows a good accuracy without concerning the time constant of the filter. Furthermore, Figure 5 shows the transition of the estimated parameters. It is verified that the addition of the disturbance on the control signal is effective for the estimation during the stabilizing control.

Table1: Basic parameter of inverted pendulum model.

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
0.1125	0.0606	0.156	0.03	0.01	0.1

Table2: Simulation condition for recursive estimation.

weights of performance index	$Q = diag[1\ \ 1\ \ 100\ \ 1]$, $r = 1$
optimal feedback gain	$F = [-0.98\ \ -1.5\ \ 35.9\ \ 6.61]$
time constant of filter G_0	$t_f = 0.001$ to $t_f = 0.03$ [sec]
sampling period	$T = 0.001$ [sec]
data numbers for estimation	$k = 10000$

Table3: Result of estimation during the control.

τ_f	Estimation error [%]					
	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\sigma}_3$	$\hat{\sigma}_4$	$\hat{\sigma}_5$	$\hat{\sigma}_6$
0.01	0.0	0.16	0.13	3.30	0.0	0.0
0.02	0.08	0.82	0.77	3.30	0.0	0.0
0.03	0.08	2.00	1.90	3.30	0.0	0.0

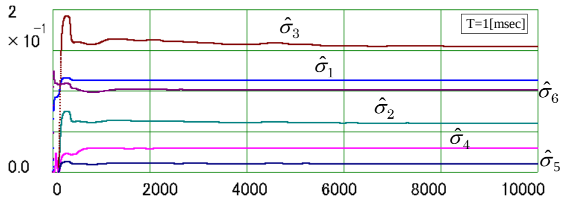

Figure 5 Transition of system parameters ($\tau_f = 0.02$).

6. EXPERIMENT OF INVERTED PENDULUM

The estimation accuracy is important because it decides the performance after tuning the controller of system. The experiment is started by the optimal regulator using the initial parameters of Table4 that estimated by means of the downward movement condition of the pendulum. In the regressor, only both signs of ϕ_{13} and ϕ_{23} change as

$$\boldsymbol{\Phi}^T(\theta,\dot{\theta},\ddot{\theta}) := \begin{bmatrix} \ddot{\theta}_1 & \phi_{12} & -\phi_{13} & \dot{\theta}_1 & 0 & \text{sgn}(\dot{\theta}_1) \\ 0 & \phi_{22} & -\phi_{23} & 0 & \dot{\theta}_2 & 0 \end{bmatrix}. \quad (21)$$

The optimal gain $\text{F} = [\,-0.98\ \ -1.47\ \ 30.3\ \ 3.33\,]$ is designed by the following performance weights:

$$\text{Q} = diag[1\ \ 1\ \ 100\ \ 1]\,, \qquad r = 1 \qquad (22)$$

After the inverted pendulum becomes upright, the recursive estimation is executed under the same torques as (20).

Table4: initial parameters estimated in the downward.

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
0.15	0.028	0.18	0.064	0.01	0.004

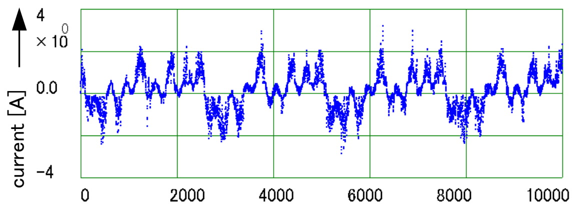

Figure 6 Actuating value appended the disturbance.

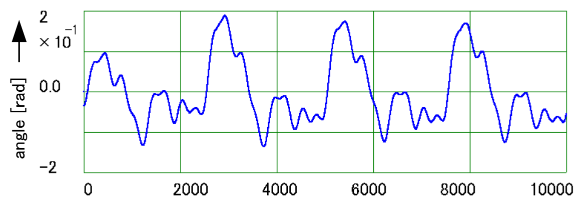

Figure 7 Angle of inverted pendulum.

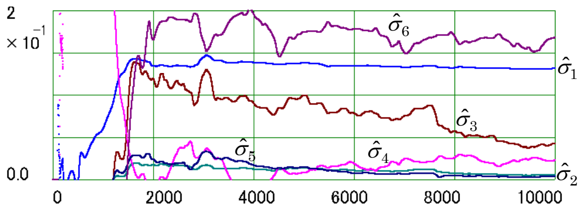

Figure 8 Transition of system parameters ($\tau_f = 0.045$).

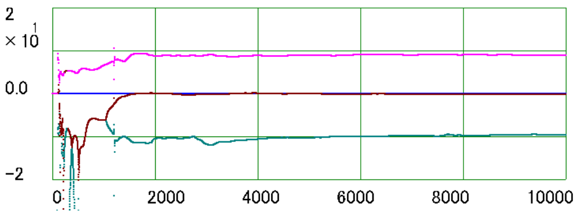

Figure 9 Transition of system eigenvalues.

Figure 6 is the actuating value appended the disturbance of (20) to the control signal. Figure 7 shows the angle of the inverted pendulum. Quite a fluctuation of the parameter appears to the estimation of the Figure 8 because of the influence of the noise and the uncertainty. Therefore, it is difficult to decide the timing for updating the controller in the tuning system. However, the eigenvalues shown in Figure 9 have the good convergence characteristics of 2 [sec]. The convergence concerning the estimated parameter can be verified by using the eigenvalues. They are quickly calculated by means of both algorithms of Householder's method using the elementary Hermitian transformation and QR method using the unitary matrix of Givens [5],[6] .

7. SELF-TUNING CONTROL

The both parameters $\hat{\sigma}_2$ and $\hat{\sigma}_3$ are important in the case of changing the pendulum length. The estimation fixing all parameters concerning the friction coefficient to the constant deteriorates. Figure 10 shows the eigenvalue in the case of fixing $\hat{\sigma}_1$ and $\hat{\sigma}_6$ to the constant. This case has better convergence speed compared with Figure 9. The estimation method reducing the possible parameter is effective in the self-tuning control. The experiment is executed by means of the mentioned strategy above in same condition as Chapter 6. After the inverted pendulum becomes upright, its length is extended. Executing the estimation simultaneously, the controller is redesigned after the instant of the convergence in the eigenvalues

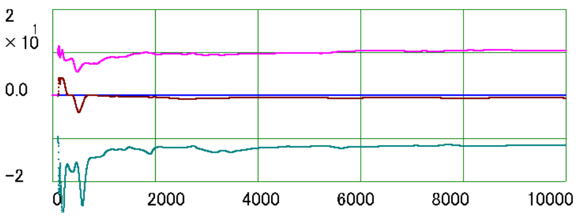

Figure 10 Transition of eigenvalues (fixing $\hat{\sigma}_1$ and $\hat{\sigma}_6$).

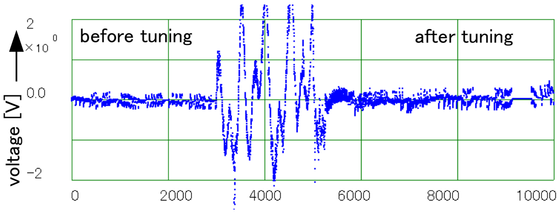

Figure 11 Actuating value with self-tuning control.

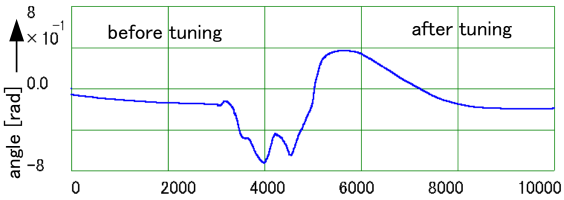

Figure 12 Angle of rotational arm.

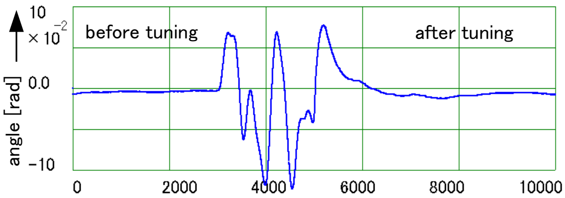

Figure 13 Angle of inverted pendulum.

calculation. Figure 11 is the actuating value appended the disturbance of (20) to the control signal during limited period of the estimation. Figure 12 is the angle of rotational arm. Figure 13 shows the angle of the inverted pendulum. A good regulation in both figures 12 and 13 appears to other parts except a short tuning interval.

8. CONCLUSIONS

The self-tuning strategy concerning the stabilized control system of the rotational inverted pendulum was achieved by considering the eigenvalues successfully.

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