Abstract: The proposed system is realized by appending the disturbance observer to the type-2 servo in order to reduce the tracking error of trajectory control. Furthermore, the adaptive control is applied to control the deterioration of the tracking performance by the variation of payload. It is verified by the practical experiment that the adaptive control is very useful in the case of the nonlinear torque has small effect compared to the friction torque.

Key words: direct drive manipulator   adaptive motion control   disturbance observer

1. Introduction

This paper reports the result of the adaptive motion control based on both type-2 servo and the disturbance observer on a two-link D.D manipulator shown in Figure 1. The recursive estimation occupies an important part to realize the adaptive control system. The estimated value is efficiently updated by means of the recursive algorithm of the scalar type effectively selected from linear relation of the vector type.

Both links of the SICE standard D.D manipulator have the same length of 200 [mm]. They are driven by the outer rotor type direct drive A.C motors in torque control mode. The maximum voltage of the torque command to both joints are ±8 volts and the maximum torque to the actuators of first and second joint are 70 [Nm] and 15 [Nm] respectively. The resolution of the optical position encoders for the first and second link is 614400 and 507904 [pulse/rev] respectively. The variations of the payload is provided by changing the weight of steel disk on the second link. The adaptive controller is implemented by using DELL Dimension V400c (Intel celeron processor 400MHz) with interface board providing 2-channel 12 bits D/A for torque command, 2-channel 24 bits 2-phase encoder pulse counter for position detection and 4-channel 12 bits A/D for current and velocity detection.

2. Manipulator dynamics

The vector equation of motion of such manipulator can be written in the general form

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q) + G(q) = \tau, \quad q \in \mathbb{R}^n, \]  

where \( \tau \) is the vector of joint torques by the actuators, \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, and \( q, \dot{q} \) and \( \ddot{q} \) are the joint angles, velocities and accelerations, respectively. The vector \( C(q, \dot{q}) \dot{q} \) represents torques arising from centrifugal and Coriolis forces. The vectors \( F(\dot{q}) \) and \( G(q) \) represent the friction torques acting at rotational joints, and the gravitational torques respectively. In the case of neglecting the friction term of (1), the dynamic equation can be usually separated with respect to the link parameters by the property of linear parameterize as

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \Phi(q, \dot{q}, \ddot{q})\sigma, \]  

where \( \Phi(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times p} \) is a signal matrix called the regressor and \( \sigma \in \mathbb{R}^p \) is the vector of link parameters.

2.1 Dynamics of the SICE D.D arm

Figure 2 shows the schematic drawing of D.D arm. In the case of horizontal plane arm setting, the gravitational term \( G(q) \) in (2) is identically zero. When the momentums of inertia concerning the links are symbolized as \( I_1, I_2 \), the terms in the dynamic equation can be obtained by means of the Euler-Lagrange dynamic equation as follows:

\[
M(q) = \begin{bmatrix}
J_1 + J_2 + 2rC_2 & J_2 + rC_2 \\
J_2 + rC_1 & J_2
\end{bmatrix},
\]  

where \( J_1 = I_1 + m_1a_1^2 + m_2\ell_1^2, \quad J_2 = I_2 + m_2a_2^2, \quad r = m_2\ell_1a_2, \quad C_1 = \cos q_1, \quad q = [q_1 \quad q_2]^T \).
are given by

Here, even though the characteristic of friction is very complicated, the approximation of friction torque is usually assumed by the following form:

where $B\dot{q}$ and $D(\dot{q})$ represent the viscous friction and Coulomb friction, respectively, as follows:

Therefore, the dynamic equation can be described by the linear parameterize form

where joint torque is $\tau_1 = \tau_2$, the regressor and the parameter vector are given by

and $\Phi(q,\dot{q}) = [\Phi_1 \Phi_2]^T$:

\[
\Phi(q,\dot{q},\ddot{q}) = \begin{bmatrix}
\dot{q}_1 & \dot{q}_2 & \Phi_{13} & \dot{q}_1 & 0 & \text{sgn}(\dot{q}_1) & 0 \\
0 & \dot{q}_1 & \dot{q}_2 & 0 & \Phi_{23} & 0 & \text{sgn}(\dot{q}_2)
\end{bmatrix},
\]

and

\[
\sigma := \begin{bmatrix}
\sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \sigma_5 & \sigma_6 & \sigma_7
\end{bmatrix}^T = [J_1 J_2 r b_1 b_2 d_1 d_2]^T. \tag{9}
\]

### 3. Manipulator identification

Here, in order to apply the update law of a scalar type, the linear parameterization (7) of vector type is appropriately separated by the following scalar forms

where the vector $\Phi_i \in R^{k_i}$ ($k_i < p$) is selected from the $i^{th}$ row vector in the regressor and the parameter vector $\theta_i \in R^{k_i}$ is properly picked up from the basic parameter $\sigma$. The estimate model is also defined as

\[
y_i(t) = \Phi_i^T(t) \hat{\theta}_i(t), (i = 1, 2), \tag{11}
\]

where the vector $\hat{\theta}_i(t) \in R^{k_i}$ is the estimated value and $y_i(t)$ is the output of the estimate model. The condition of the recursive estimation is completed in every sampling interval by filling the motion data

\[
\{ q_i(t), \dot{q}_i(t), \ddot{q}_i(t), \tau_i(t) \}, (i = 1, 2). \tag{12}
\]

When the condition in signal to noise ratio of the dynamical system is good, the following least-squares estimation has small bias with respect to the estimated value.

**Recursive least-squares algorithm:**

\[
\hat{\theta}_i(t) = \hat{\theta}_i(t - 1) + \frac{P_i(t - 1)\Phi_i(t)}{\lambda_i(t) + \Phi_i^T(t)P_i(t - 1)\Phi_i(t)} e_i(t),
\]

\[
P_i(t) = \frac{1}{\lambda_i(t)} \left[ P_i(t - 1) - \frac{P_i(t - 1)\Phi_i(t)\Phi_i^T(t)P_i(t - 1)}{\lambda_i(t) + \Phi_i^T(t)P_i(t - 1)\Phi_i(t)} \right],
\]

\[
e_i(t) = \tau_i(t) - \Phi_i^T(t) \hat{\theta}_i(t - 1),
\]

\[
\lambda_i(t) = (1 - \mu_i)\lambda_i(t - 1) + \mu_i, \quad (i = 1, 2), \tag{13}
\]

where weighting sequences $\lambda_i(t)$ is usually chosen within $0.98 < \lambda_i(t) \leq 1$ and $\mu_i$ is the constant for adjusting (13).

### 3.1 The regressor for identifying D.D arm

There are some ways for separating the regressor vector in identifying basic parameters of D.D arm. It is necessary to arrange these parameters so that a separated system does not mutually have an influence upon the apriori error $e_i(t)$. The pairs of identification model with respect to regressor and estimate vector satisfying such conditions are given by

\[
\Phi_1^T(t) = [\dot{q}_1(t) \ \dot{q}_1(t) \ \text{sgn}(\dot{q}_1(t))]
\]

\[
\Phi_2^T(t) = [\dot{q}_2(t) + \dot{q}_2(t) \ \phi_{21}(t) \ \phi_{22}(t) \ \text{sgn}(\dot{q}_2(t))]
\]

\[
\Theta_1^T(t) = [\dot{\sigma}_1(t) \ \dot{\sigma}_2(t) \ \dot{\sigma}_3(t) \ \dot{\sigma}_4(t) \ \dot{\sigma}_5(t) \ \dot{\sigma}_6(t) \ \dot{\sigma}_7(t)]. \tag{14}
\]

### 4. Acquisition of regressor

When the system identification has different sensor dynamics, the accuracy of estimation deteriorates. The availability of the filter for compensating the difference of sensor characteristics was proposed in$^{[10]}$. However, the consideration of observation noise was not fully verified. When the estimation accuracy is considered on data acquisition, the compensation scheme$^{[11]}$ of sensor dynamics shown in Figure 3 is profitable compared to the accessible case of only joint torque and position. The link acceleration is calculated by using the derivation filter to the information of the velocity sensor. The motion data of the estimated acceleration, torque, position and velocity, etc. is given, respectively, by the filters as follows:

\[
\tau_x = G_0 \tau, \quad q_x = G_0 q, \quad \text{sgn}(\dot{q}_x) = G_0 \text{sgn}(\dot{q})
\]

\[
\dot{q}_x = G_0 \dot{q}, \quad \dot{q}_x = G_0 \ddot{q}
\]

\[
G_0 = \frac{1}{1 + \tau_x(s)}, \quad G_1 = sG_0. \tag{15}
\]

Figure 2  Schematic drawing of two-link D.D manipulator.

Figure 3  Schematic drawing of two-link D.D manipulator.
respectively, by following:

The reference and disturbance characteristics are given, respectively. Then, the resultant transfer function of the scalar system as type-2 servo shown in Fig.4 is designed by using the approximate model of the scalar system as

\[ G(T) = \frac{1}{2T(z-1)} \]

Neglecting the non-diagonal element of inertia matrix, torque does not appear so much during trajectory control.

5. Control system for D.D arm

The friction torque and Coulomb torque of the SICE standard D.D manipulator are large compared to the nonlinear torque. The effect of the centrifugal and Coriolis torque does not appear so much during trajectory control.

5.1 Type-2 servo system

Neglecting the non-diagonal element of inertia matrix, type-2 servo shown in Fig.4 is designed by using the approximate model of the scalar system as

\[ J \dot{q} + B \dot{q} = \tau \]

D.D manipulator \( P_s \) is described as

\[ P_s = PG_s, \quad P = 1/(JS + B), \quad G_s = K_s/s \]

where \( K_s \) is the conversion constant of radian to degree. Furthermore, \( G_d \) represents the servo driver

\[ G_d = K_i/(1 + T_i s) \]

where \( K_i \) and \( T_i \) represent the gain and time constant of torque amplifier, respectively. Then, the resultant transfer function of \( G_d P_s \) is described as

\[ G_d P_s = b/(s^2 + a_s s + a_2) \]

where \( a_i = (J + T_i B)/T_{Ji}, a_2 = B/J_{Ti}, b = K_i K_{ij}/J_{Ti} \).

When the characteristic polynomial is specified by \((s+\omega)^6\), type-2 servo controller \( G \) is given by

\[ G = (a_3 \omega^3 + a_2 \omega^2 + a_1 \omega + 1)/(s^2 + \delta_1 s + \delta_2) \]

\[ \delta_1 = 6\omega - a_s \omega - a_2 \delta_2 - a_1 \delta_3 
\]

\[ \delta_2 = 15\omega^2 - a_s \delta_2 - a_2 \delta_3, \quad \delta_3 = 5a_1 \]

\[ b_1 = (20\omega^3 - a_3 \delta_2 - a_1 \delta_3)/b \]

\[ b_2 = 6\omega^2/b, \quad b_3 = \omega^4/b, \quad \tau_p = 6/\omega \]

The reference and disturbance characteristics are given, respectively, by following:

\[ \tau = M(q) \dot{q} + C(q, \dot{q}) \dot{q} + B \dot{q} + D(q) \]

The operation of these filters is also calculated in discrete form using the bilinear transformation:

\[ s = \frac{2}{T} \left( \frac{z-1}{z+1} \right), \quad T: \text{sampling period.} \]

5.2 Disturbance observer

The modeling error is compensated by appending the disturbance observer to type-2 servo. The disturbance observer shown in Fig.5 estimates the modeling error and parameter variation, etc. as one disturbance variable. If the payload variation is not to large, the control system by the feedback of this variable has robustness and is one of useful methods. The low-pass filter \( Q_1 \) and \( Q_2 \) are specified by

\[ Q_1 = \frac{1}{1 + \tau_i Q_1}, \quad Q_2 = \frac{1}{1 + \tau_i Q_2} \]

Then, the reference characteristic is similar to (24), the disturbance characteristic is given by

\[ q = (1 - Q_1 Q_2)/(1 - G P_s) d \]

Therefore, the sensitivity is reduced to \((1 - Q_1 Q_2)\) times as compared with type-2 servo.

5.3 Adaptive control system

The control performance deteriorates in the case of operating with the variation of payload, because the servo controller and the disturbance observer depend on the manipulator payload. The proposed adaptive control system shown in Fig. 6 is proposed by appending the recursive estimation to disturbance observer system. This system updates both servo controller and disturbance observer by estimating the payload variation of D.D manipulator.

6. Trajectory control results of D.D arm

The nominal parameter shown in Table 1 is obtained by off-line identification. Each gain \( K_i \) of joint is, respectively, 12.5 and 3.75 [Nm/V].
The time constant of torque amplifier is $T = 0.009$. Type-2 servo controllers for each joint are designed by selecting the specified pole of $\tau_p = 0.1$. The circular orbit of diameter 100[mm] is given as reference trajectory. A period of movement is 2[sec], since sampling period is 1[ms] and also data of reference angle to each joint is selected as 2000.

The tracking error from a circular orbit by trajectory control experiments with a light payload is shown in Figure 7. The large errors due to disturbance torque appear periodically. When the time constants of low-pass filter for disturbance observer and the pole of servo controller are selected, respectively, as $T_1 = 521[\text{ms}]$ and $T_2 = 0.994[N\text{m}]$, the error with same condition is shown in Figures 8 and 9. The error is effectively decreased by using the disturbance observer.

However, when same experiment is repeated by changing only payload to 3[kg], Figures 10 and 11 reveal that the control system is not satisfactory and loses the robustness. Therefore, it is shown in Figures 12 through 14 that the adaptive control is very useful. Figure 12 is the transition process of recursively estimated parameter with filter time constant of $T_f = 6[\text{ms}]$ and the parameter estimation completes after passing 2[sec]. The adaptive system starts the adjusting of both controller and disturbance observer after passing 2[sec]. After starting the parameter adjusting, the vibration of D.D robot disappears and the tracking error is simultaneously decreased as shown in Figures 13 and 14.

### 7. Conclusions

The adaptive control system appending the disturbance observer to the type-2 servo in order to reduce the tracking error of the trajectory control was proposed. The validity of the proposed method was confirmed by applying to the SICE D.D robot.

### References