# Adaptive Controller Design for Anti-Slip System of EV

Phornsuk RATIROCH-ANANT

Dept. of Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang Bangkok, Thailand ktphorns@kmitl.ac.th

Abstract—The design strategy of adaptive controller for antislip system of EV (electric vehicle) is proposed. The proposed system is constructed by preparing both systems of disturbance observer and parameter estimation. The load variation of the tire slip is observed as the equivalent disturbance of motor system by employing the adaptive disturbance observer, and the vehicle speed is estimated. The tire slip is appropriately controlled by means of the feedback of the speed deviation between wheel speed and estimated vehicle speed. It is proved by the numerical simulations that the proposed methods provide satisfactory performance. Furthermore, the practical DC motor experiment set, which can give various slips, is constructed in order to investigate the effect of proposed anti-slip control system. It is verified that the proposed strategy is useful as one approach of anti-slip control.

*Keywords*—anti-slip system, electric vehicle, adaptive control, disturbance observer, DC-motor

# I. INTRODUCTION

The studies [1][2] concerning the traction control recently become active recently in the field of vehicle dynamic behavior. Above all, both modeling of tire characteristics and new control strategy are important in development of electric vehicle. The tire slip of the vehicle occurs by both radical acceleration and braking depending on the condition of the road surface. There are several reports [3][4] concerning such traction control that can place the slip ratio within the suitable range as the prevention strategy of the slip. However, few design methods consider system parameter variation system including such uncertain parameters as vehicle weight and so on, concerning the vehicle system. In this paper, the controller design for the anti-slip of the vehicle, which have unknown motion parameters is achieved by the adaptive control introducing the disturbance observer [3][5]. Assume the tire slip, the vehicle weight and the momentum of inertia, which are unknown. Then, the performance of the recursive estimation with respect to both system parameter and disturbance is verified by using numerical simulation. The so-called Magic Formula [6][7] that is widely used as standard model of road surface friction in the automobile engineering is employed for the simulation.

Furthermore, the experiment set is constructed by means of the simple model that is the roller is connected to DC motor to Hiroshi HIRATA, Masatoshi ANABUKI and Shigeto OUCHI **Dept. of Applied Computer Engineering** Tokai Univ., Hiratsuka, Japan hirata@keyaki.cc.u-tokai.ac.jp

drive the belt of the belt-pulley mechanism that causes various slips. The effect of proposed anti-slip control system is investigated. It is proved by experiment result that the proposed method is useful as one approach for anti-slip control of unknown parameter system.

#### II. ONE WHEEL VEHICLE MODEL

A one-wheel vehicle model as shown in Fig.1 is constructed, which consists of the wheel system and the vehicle body system. The wheel system is directly driven by DC motor and its motion equation is described as follows:

$$L\frac{di}{dt} + Ri = K_p e_i - K_e \omega$$

$$J\frac{d\omega}{dt} + B\omega = K_\tau i - Fr - D\operatorname{sgn}(\omega)$$
(1)

Assume the slip ratio  $\lambda$  between tire and road surface is defined by

$$\lambda = \frac{r\omega - v}{r\omega} , \text{ with } r\omega > v \text{ of driving,}$$
(2)

the motion equation of the vehicle body system is also described by

$$\begin{cases} M \frac{dv}{dt} + Cv = F \\ F = \mu N, \quad \mu = f(\lambda) \end{cases},$$
(3)



Figure 1. One-wheel vehicle model.

the coefficient of road surface friction  $\mu$  of the vehicle body system is described by the nonlinear function of slip ratio  $\lambda$ .

## III. ADAPTIVE CONTROLLER DESIGN

The proposed control system observes the load change of the wheel-slip as the equivalent disturbance by introducing the disturbance observer and also provides the estimation system of the motion parameters because in general the vehicle weight sometimes changes in general. Moreover, the vehicle speed is derived by using both the observed disturbance and the estimated parameters.

#### A. Slip detection and estimation of vehicle speed

Assume the drive torque and Coulomb friction torque are considered as disturbance torque as following:

$$\tau_f = Fr + D\operatorname{sgn}(\omega) \qquad (4)$$

Neglecting the variation of the disturbance  $\dot{\tau}_f \equiv 0$  in order to introduce minimal order observer, the system is described by the following state equation:

$$\frac{d}{dt} \begin{bmatrix} \tau_f \\ i \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -R/L & -K_e/L \\ -1/J & K_\tau/J & -B/J \end{bmatrix} \begin{bmatrix} \tau_f \\ i \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ K_p/L \\ 0 \end{bmatrix} e_i \quad (5)$$
$$\begin{bmatrix} i \\ v_\omega \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} \tau_f \\ i \\ \omega \end{bmatrix}. \quad (6)$$

Moreover, the such observer pole is specified as  $a = 1/T_2$ , the disturbance observer system is given by

$$\begin{cases} z = -az + aK_{\tau}i + a(aJ - B)\omega\\ \hat{\tau}_f = z - aJ\omega \end{cases}.$$
(7)

In addition, Equation (8) is obtained from (1) and (7).

$$\frac{d\hat{\tau}_f}{dt} = \frac{dz}{dt} - aJ\frac{d\omega}{dt} = -a\left(\hat{\tau}_f - \tau_f\right)$$
(8)

Therefore, asymptotic stability of the disturbance observer is ensured. Considering both observation  $\hat{\tau}_f$  and (3), the vehicle speed *v* is given by the following formulation:

$$\frac{d\hat{v}}{dt} = -\frac{C}{\hat{M}}\hat{v} + \frac{1}{\hat{M}r}(\hat{\tau}_f - D\operatorname{sgn}(\omega)).$$
(9)

# B. Control system

The following current amplifier

$$\begin{cases} T \dot{e}_i + e_i = K_1 e\\ e = u - K_i R_i i \end{cases},$$
(10)

is usually used for torque control of DC motor. However, in order to promote easier system analysis we will discuss the system stability by simplifying the current amplifier as

$$\begin{cases} i = K_a u \\ J\dot{\omega} + B\omega = K_\tau i - \tau_f \end{cases}$$
(11)

Here, assume the manipulated variable u is given by

$$u = u_r - \rho$$
, with  $\rho = v_\omega - \hat{v}$ . (12)

Considering the condition  $u_r \equiv 0$  and using estimation  $d\hat{v}/dt$  in (9), the following equation is obtained from (11) and (12):

$$\frac{d\rho}{dt} = \frac{dv_{\omega}}{dt} - \frac{d\hat{v}}{dt} = r\frac{d\omega}{dt} - \frac{d\hat{v}}{dt}$$

$$= r\left\{-\frac{B}{J}\omega + \frac{K_{\tau}K_{a}}{J}(\hat{v} - v_{\omega}) - \frac{1}{J}\tau_{f}\right\} - \left\{\frac{1}{\hat{M}r}\hat{\tau}_{f} - \frac{C}{\hat{M}}\hat{v}\right\}$$

$$= -\left\{\frac{B}{J} + \frac{K_{\tau}K_{a}r}{J}\right\}v_{\omega} + \left\{\frac{C}{\hat{M}} + \frac{K_{\tau}K_{a}r}{J}\right\}\hat{v} - \left\{\frac{r}{J}\tau_{f} + \frac{1}{\hat{M}r}\hat{\tau}_{f}\right\} \quad (13)$$

Furthermore, considering the observer stability  $\hat{\tau}_f = \tau_f$  and such assumption that the friction coefficients of *B* and *C* are smaller than both vehicle weight *M* and motor inertia *J*, the relation of speed error  $\rho$  is given by

$$\frac{d\rho}{dt} = -\frac{K_{\tau}K_{a}r}{J}\rho - (\frac{1}{\hat{M}r} + \frac{r}{J})\tau_{f} \quad . \tag{14}$$

The effect of anti-slip  $\rho \rightarrow 0$  can be expected because the completion of  $\tau_f \approx 0$  is evident at remarkable slip.

#### C. Recursive parameter estimation system

Equation (9) is useful for the estimation of the vehicle speed v. However, the vehicle weight M, which changes about 50 percent at the worst case, is necessary in calculation. Therefore, the parameter estimation system is appended to the disturbance observer. If the slip does not occur, the following equation is obtained from (1) and (3):

$$(J + Mr^2)\frac{d\omega}{dt} + (B + Cr^2)\omega = K_{\tau}i - D\operatorname{sgn}(\omega).$$
(15)

The motion parameters such as  $J + Mr^2$ ,  $B + Cr^2$  and D are estimated in advance because the linear estimation is applicable to the following estimation model:

$$y_e(t) = \varphi_e^T(t) \hat{\theta}_e(t) , \qquad (16)$$

$$\varphi_e^T(t) = [\dot{\omega}(t) \quad \omega(t) \quad \operatorname{sgn}(\omega(t))], \ \hat{\theta}_e(t) = [\hat{\theta}_1(t) \quad \hat{\theta}_2(t) \quad \hat{\theta}_3(t)]^T.$$
(17)

Therefore, assume the parameter of Coulomb friction D is known hereafter. In case of considering the slip, the following equation is obtained from (1) and (4):

$$J\dot{\omega} + B\omega = K_{\tau}i - \tau_f \quad . \tag{18}$$

Here, the following estimation model

$$\boldsymbol{\tau}(t) = \boldsymbol{\varphi}^T(t)\hat{\boldsymbol{\theta}}(t) + \hat{\boldsymbol{\tau}}_f \tag{19}$$

is implemented in the estimation system shown in Fig. 2 and  $\hat{\tau}_f$  means the disturbance observer output of (7).



Figure 2. Estimation system connected to disturbance observer.

The drive torque  $\tau_m$  and the motor speed  $\omega$  are measured by the current detector  $R_i$  and the optical encoder, respectively. The necessary motion data  $\varphi^T(t) = [\dot{\omega}_a(t) \ \omega_a(t)]$  and  $\tau_m(t) = \tau_a(t)$  for the estimation are given by the following low pass filters [5]:

$$\begin{cases} \tau_a = G_0 \tau_m \\ \omega_a = G_0 \omega \\ \dot{\omega}_a = G_1 \omega \end{cases}, \quad G_0 = \frac{1}{\left(1 + T_1 s\right)^2}, \quad G_1 = s G_0 \qquad (20)$$

The operation of the filters is also calculated in discrete form using the bilinear transformation:

$$s = \frac{2}{T} \left( \frac{z-1}{z+1} \right)$$
,  $T: sampling period.$  (21)

The estimation approach based on the following RLS (recursive least squares) method [8][9] is reliable and has few biases with respect to the estimation value if the condition of signal to noise (S/N) ratio of vehicle system is suitable. The prediction error of (24) is the characteristic of the proposed system, and the estimation value  $\hat{\tau}_f$  of disturbance observer is used effectively in the algorithm.

# [RLS algorithm]:

Parameter adjust:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)\varphi(t)}{\lambda(t) + \varphi^{T}(t)P(t-1)\varphi(t)}\varepsilon(t)$$
(22)

Adaptive gain:

$$P(t) = \frac{1}{\lambda(t)} \left\{ P(t-1) - \frac{P(t-1)\varphi(t)\varphi^{T}(t)P(t-1)}{\lambda(t) + \varphi^{T}(t)P(t-1)\varphi(t)} \right\}$$
(23)

Prediction error:

$$\varepsilon(t) = \tau_a(t) - \varphi^T(t)\theta(t-1) - \hat{\tau}_f$$
(24)

Weighting coefficient:

$$\lambda(t) = (1 - \mu)\lambda(t - 1) + \mu , \quad (\mu = 0.01)$$
(25)

#### D. Adaptive control system

The adaptive control system shown in Fig. 3 can be constructed by taking the disturbance observer together with recursive estimation. The estimation  $\hat{M}$  of the vehicle weight is obtained by considering the relation of (15) from the estimated



Figure 3. Block diagram of adaptive controller.

parameter of motor inertia because the estimation system is able to monitor the momentum of inertia  $\hat{J}$  of motor. Moreover, the designer is able to update the disturbance observer according to the situation of the motor system too.

# E. Approximation model of vehicle characteristics

The coefficient of road surface friction was given by the nonlinear function  $\mu = f(\lambda)$  of the slip ratio  $\lambda$  in the vehicle motion equation of (3). There are several approximations as the experimental formula that described the nonlinear function. The following Magic Formula [6][7], which is a standard model of road surface friction in the automobile engineering, is used at the snow condition:

$$\mu(\lambda) = D_m \sin \left[ C_m \tan^{-1} B_m \left\{ (1 - E_m) \lambda + \frac{E_m}{B_m} \tan^{-1} B_m \lambda \right\} \right], \quad (26)$$
  
with  $B_m = 26.66, \ C_m = 1.4, \ D_m = 0.24, \ E_m = 0.4$ .

Fig. 4 shows typical  $\mu - \lambda$  characteristics obtained by (26). The  $\mu - \lambda$  characteristics are useful in the numerical simulation of the adaptive control system in next section IV.



Figure 4.  $\mu - \lambda$  characteristics by means of Magic Formula.



Figure 5. Block diagram for adaptive control system simulation.

#### IV. NUMERICAL SIMULATION OF ADAPTIVE SYSTEM

The numerical simulation of adaptive control system with respect to Fig. 5 is executed with the condition shown in TABLE I when the command signal  $u_r$  for motor driver is the pulse input of 0 to 0.3[Volt] and the sampling period for the control system is chosen as 1 [ms]. Each constant in TABLE I is selected by considering the DC motor experiment set, although they have very small values in comparing with electrical vehicle. Both parameters of motor inertia J and viscous friction coefficient B are evaluated by recursive estimation in order to verify the performance of the adaptation system. We recursively adjust only  $\hat{J}$  without adjusting *B* because the result of both parameter estimation and observer was not greatly influenced by B of disturbance observer. The coefficient  $\mu$  of road surface friction by Magic Formula (26) is first calculated because the vehicle body speed v is necessary in order to express the slip ratio  $\lambda$ . It is next calculated by the motion equation of (3) after the drive force F was obtained by assumption of vertical force N = 0.7[N].

TABLE I. Simulation condition for anti-slip control.

momentum of rotor inertia	J = 4.00E - 5 [kgm <sup>2</sup> ]
vehicle weight, radius of wheel	M = 0.06 [kg], $r = 0.03$ [m]
coefficient of viscous friction	$B = 2.48E - 5 \qquad [Nms / rad]$
coefficient of Coulomb friction	$D = 7.84E - 3 \qquad [Nm]$
armature inductance and resistance	$L = 6.0 \ [mH], \ R = 4.5 \ [\Omega]$
feedback ratio of current	$K_i \cdot R_i = 5 \times 0.2$
back electromotive force constant	$K_e = 0.07162  [V \text{ sec}/rad]$
torque constant of motor	$K_r = 0.07154$ [Nm/A]
amplifier of both current and power	$G_i \cdot K_p = \frac{4 \times 15}{1 + 0.01s}$
observation noise of motor speed	$\sigma^2 = 1.0E - 6 \qquad [V]$



Figure 7. Transition of slip ratio by open loop system.

The viscous friction coefficient *C* of vehicle body is also ignored in the simulation because it is not essential in the proposed system. The simulation results are shown in Figs. 6 to 12. Fig. 6 shows both wheel speed  $v_{\omega}$  and vehicle body speed  $\hat{v}$  in the open-loop mode without the feedback of the speed deviation  $v_{\omega} - \hat{v}$  and Fig. 7 shows the slip ratio  $\lambda$ . The simulation result reveals that the slip is not controlled in the acceleration time.

On the other hand, the simulation results of adaptive control with the feedback of the speed deviation are shown in Figs. 8 to 12. Fig. 8 shows the drive torque  $\tau$  of motor with the command signal  $u_r$  of step pulse. Fig. 9 compares the disturbance torque  $\tau_f$  of motor with the estimation  $\hat{\tau}_f$  of disturbance observer in the situation of slip occurrence. Fig. 10 also compares both wheel speed  $v_{\omega}$  and vehicle body speed  $\hat{v}$ . Furthermore, Fig. 11 shows the slip ratio  $\lambda$  and it is suitably controlled immediately after starting the simulation. Finally, Fig. 12 shows the estimated parameter and its accuracy are placed within error rate of 3[%] under the presence of observation noise and its convergence speed is sufficient for the adaptive control too. It was confirmed through their results that the estimation accuracy of disturbance appear to be sufficient and the proposed anti-slip control is also executed appropriately.



Figure 8. Drive torque by adaptive control.



Figure 9. Disturbance and estimated disturbance.





Figure 11. Transition of slip ratio by adaptive control.



# V. ADAPTIVE ANTI-SLIP CONTROL BY EXPERIMENT SET

The performance of anti-slip control is evaluated by means of DC motor experiment set shown in Fig. 13. The roller connected to DC motor in Fig. 13 is regarded as the wheel of vehicle and the pulley with timing belt is regarded as the vehicle body. The DC motor is connected to the optical encoder of 2000 [*pulse/rev*] resolution and the constant for converting the motor speed to the analog voltage signal for its detection is  $S_v = 3.183 \times 10^{-2}$  [*V* sec/*rad*]. Moreover, another optical encoder of the resolution of 2500 [*pulse/rev*] is coupled to the pulley shaft in order to evaluate the estimated speed  $\hat{v}$  with the speed of pulley. The slip occurs between roller and timing belt, and the movable mount base adjusts the



Figure 13. Experiment set for evaluating the ant-slip control.

tension of their contact. The numerator parameter  $\hat{J}$  of the disturbance observer is recursively updated at every sampling time *T* according to the estimated parameter, although the assurance of its stability becomes not clear in case of including variable parameter in it.

The experiment is executed by the same condition as simulation that the command signal  $u_r$  for motor driver is the pulse input of 0 to 0.3[Volt] and the sampling period for the control system is chosen as 1 [ms]. The experiment results are shown in Figs. 14 to 20. Fig. 14 shows both motor speed  $v_{\omega}$  and timing-pulley speed  $\hat{v}$  in the open-loop mode without feedback of the speed deviation  $v_{\omega} - \hat{v}$  and Fig. 15 shows the slip ratio  $\lambda$ . The results reveal that the slip is not controlled in the acceleration time.

On the other hand, the experiment results of adaptive control with the feedback of the speed deviation are shown in Figs. 16 to 20.



Figure 14. Motor speed and pulley speed by open loop system.



Figure 15. Slip ratio by open loop system.



Fig. 16 shows the drive torque  $\tau$  of motor with the command signal  $u_r$  of step pulse and Fig. 17 shows the estimation  $\hat{\tau}_f$  of disturbance observer in the situation of slip

occurrence. Furthermore, Fig. 18 compares both motor speed  $v_{\omega}$  and timing-pulley speed  $\hat{v}$ . Fig. 18 simultaneously shows the variation of the slip ratio  $\lambda$  and it is suitably controlled within 1 [sec] after starting the experiment. Comparing the results of Figs. 18 and 19 with the results of Figs.14 and 15, it is clear that the occurrence of the slip is decreased by the proposed system.

Finally, from Fig. 20, even the convergent speed of the estimated parameters in the practical system is slow compared to the simulation result, but it does not have a difficulty for the adaptive control. It was confirmed through their results that the estimation speed of disturbance is sufficient and the anti-slip control is also executed appropriately.

#### CONCLUSIONS

It was proved that the proposed strategy not only represents good performance for anti-slip control of the vehicle system but also has sufficient convergence speed in the parameter estimation through numerical simulation. Furthermore, satisfactory result was also obtained by investigating the effect of anti-slip control in the DC motor experiment set. The adaptive control system, which provides both disturbance observer and parameter estimation, is useful for such parameter variation system as vehicle control. Furthermore, the accuracy of the parameter estimation should be improved in the practical use.

#### REFERENCES

- Toshimichi Takahashi, "Modeling, Analysis and Control Methods for Improving Vehicle Dynamic Behavior (Overview)," R&D Review of Toyota CRDL, Vol.38, No.4, 2003, pp.1-9.
- [2] Yoshikazu Hattori, "Optimum Vehicle Dynamics Control Based on Tire Driving and Braking Forces," R&D Review of Toyota CRDL, Vol.38, No.4, 2003, pp.23-29.
- [3] Y. Hori, Y. Toyoda and Y. Tsuruoka, "Traction control of electric vehicle: basic experimental results using the test EV," IEEE Trans. on Industrial Application, Vol.34, No.5, 1998, pp.1131-1138.
- [4] S.Sakai, H.Sado and Y.Hori, "Motion control in an electric vehicle with 4 independently driven in-wheel motors," IEEE Trans. on Mechatronics, Vol.4, No.1, 1999, pp.9-16.
- [5] P.RATIROCHANANT, M.Anabuki and H.Hirata, "Adaptive Motion Control of a Two-Link Direct Drive Manipulator using Disturbance Observer," Proc. Of IEEE Region 10 Conference On Computers, Communications, Control And Power Engineering (TENCON'02), Beijing, 2002, pp.1725-1728.
- [6] H.B.Pacejka and E.Bakker, "The Magic Formula Tyre Model, In Proc. 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis," Delft, Netherlands, Vol.21, 1991, pp.1-18.
- [7] M. Mizuno., et al., "The Magic Formula Tyre Model Using the Measured Data of a Vehicle Running on Actual Roads," Proc. of 4th Int. Symp. Advanced Vehicle Control (AVEC'98), Nagoya, 1998, pp.329-334.
- [8] I.D.Landau, System Identification and Control Design, Prentice Hall, 1990.
- [9] H.Hirata, P. Ratiroch-anant and M. Anabuki, "Auto-Tuning for Strongly Stable Adaptive Pole Placement Control System," Proc. Of The 4<sup>th</sup> Asian Control Conference, Singapore, 2002, pp.507-512.