1999 IEEE TENCON

ACHIEVING STRONGLY STABLE ADAPTIVE CONTROL SYSTEM USING INTELLIGENT AUTO-TUNING

Phornsuk RATIROCH-ANANT *, Vipan PRIJAPANIJ *, Jongkol NGAMWIWIT *
Masatoshi ANABUKI ** and Hiroshi HIRATA **,

* Faculty of Engineering and Research Center for Communications and Information Technology,
King Mongkut's Institute of Technology, Ladkrabang, Bangkok 10520, THAILAND
E-mail:ktphorns@kmitl.ac.th

** Department of Control Engineering, School of Engineering, Tokai University,
1117 Kitakaname, Hiratsuka, Kanagawa 259-1292, JAPAN
E-mail: hirata@keyaki.cc.u-tokai.ac.jp

Abstract

The practical design method of a strongly stable system for the adaptive control is proposed. The stable pole of an optimal servo system is specified to the pole placement of a closed-loop in the adaptive control system and the stability index is introduced for the evaluation of the relative stability. The appropriate characteristics can be derived, by adjusting automatically the weight of a performance index in optimal control by means of the fuzzy inference on the basis of a stability index. Numerical simulation is used to prove that the proposed method provides sufficient performance regarding both tuning and control.

I. INTRODUCTION

In industry fields, there are many control systems that the dynamic characteristics of controlled process change greatly. For example, the parameter of mechanical system on motor control field changes over a wide range. Furthermore, the speed control system on the rolling lines of steel mills is a plant whose load inertia fluctuation is very large. Accordingly, control performance usually deteriorates when applying many conventional control methods with fixed parameter as controller. Therefore, that an adaptive control [1], [2], is a very effective method for such systems is known. Recently, the realization of intelligent auto-tuning on the motor control system is tried actively.

By the way, when the system parameters change over a wide range, a series compensator having an unstable pole appears in the adaptive pole placement control system frequently, even if the closed-loop was designed as a stable system. The appearance of the unstable compensator is not desirable with respect to both stability and reliability. The unstable controller is reluctantly used, especially if the plant itself is a stable system. Therefore, in order to obtain a stable controller over the full range of parameter change, appropriate selection of the pole of a closed-loop system is required. However, selecting the closed-loop pole ensuring the stability of compensator under condition of parameter change over a wide range is very difficult in the pole placement control. In other words, in practice it is easily proved that designer cannot select this closed-loop pole as the fixed constant value. Therefore, the construction of a strongly stable system [3] is facilitated if designer is able to adjust the stable poles of the closed-loop system recursively according to the change of plant parameter. Such research is an indispensable significant theme to the development of intelligent auto-tuning technology. Recently, authors announced an effective design method [4] that constructs a strongly stable system with respect to the adaptive pole placement, even if the plant parameter changes greatly. This presented method does not only ensure the stability of both closed-loop system and controller but also can achieve fine control performance. Furthermore, That in an adaptive pole placement system the design method evaluates by introducing a stability index [5] the relative stability of both series compensator and closed-loop system is main characteristic. An optimal servo system [6] is recursively designed to an estimated plant model in this approach. The series compensator is automatically constructed by solving *Bezout* identity on the basis of the characteristic polynomial of an optimal servo system. New resultant index is selected as the scaling factor for fuzzy inference [7] from the stability indices of series compensator in order to evaluate relation between the stability of a controller and the performance of a closed-loop system and simultaneously it can be related to the weight of performance index in optimal control. By considering these relations, appropriate tuning system of the performance weight based on relation between stability index and membership function is automatically achieved.

II. ADAPTIVE CONTROL SYSTEM

It is difficult in many discrete-time systems really to avoid an unstable zeros. The adaptive pole placement control system adjusts automatically the control system by repeating the estimation and the control alternately. Therefore it has the characteristic that the structure of estimation and control are separating. Furthermore, the designer need not consider the problem of the unstable zeros because this system does not use unstable pole-zeros cancellation.

II-1 Control System

The adaptive control system of this research is constructed by means of the pole placement control system introduced with an integrator in order to reject stationary disturbance. The plant is given by ARX system described according to the following:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + w(k) \ , \tag{1}$$

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n} \ , \tag{2}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_mq^{-m} \ , \tag{3}$$

where $w(k)$ is the white noise having zero mean value. The reference signal is generated in accordance with the following:

$$A_m(q^{-1})r_m(k) = q^{-d}B_m(q^{-1})u_m(k) \tag{4}$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + \cdots + a_{ml}q^{-l} \ , \tag{5}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \cdots + b_{ml}q^{-l} \ . \tag{6}$$

At this time, the regulation performance of stable closed-loop system is specified by the following polynomial:

$$D_o(q^{-1}) = 1 + d_1q^{-1} + \cdots + d_{nd}q^{-nd} \ , \ nd \le n + m + d \ . \tag{7}$$

In addition, the tracking performance of control system is generally achieved using prefilter $T(q^{-1})$, either the constant such as

$$T(q^{-1}) = D_o(1)/B(1) \tag{8a}$$

or the polynomial such as

$$T(q^{-1}) = D_o(q^{-1})/B(1) \ . \tag{8b}$$

Here, let us consider the error described by

$$e(k+d) = D_o(q^{-1})y(k+d) - T(q^{-1})B(q^{-1})r_m(k+d) \ . \tag{9}$$

The variance of (9) is defined as the performance index:

$$J = E[e^2(k+d)] \ . \tag{10}$$

Then, the optimal actuating value $u(k)$ that minimizes J is constructed by means of the following signal:

$$u(k) = \frac{T(q^{-1})r_m(k+d) - R(q^{-1})y(k)}{(1-q^{-1})S(q^{-1})} \ , \tag{11}$$

where $S(q^{-1})$ and $R(q^{-1})$ are given by the polynomials:

$$S(q^{-1}) = 1 + s_1 q^{-1} + \cdots + s_{ns} q^{-ns} \ , \quad ns = m + d - 1 \ , \tag{12}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \cdots + r_{nr} q^{-nr} \ , \quad nr = n \ . \tag{13}$$

Furthermore, $S(q^{-1})$ and $R(q^{-1})$ can be derived by solving the *Bezout* identity (or the *Diophantine* equation):

$$D_o(q^{-1}) = (1-q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) \ . \tag{14}$$

Ⅱ-2 Identification System

When the parameter of ARX system (1) is unknown or greatly changeable, the estimated value of a parameter is substituted in the control system . If the parameter vector θ and the regression vector $\varphi(k)$ constructed by measurement data are defined as

$$\theta^T = \{a_1, a_2, \cdots, a_n, b_0, b_1, \cdots, b_m\} \tag{15}$$

and

$$\varphi^T(k) = \{-y(k-1), -y(k-2), \cdots, -y(k-n), \; u(k-d), u(k-1-d), \cdots, u(k-m-d)\} \ , \tag{16}$$

respectively, then the plant output $y(k)$ is expressed as follows:

$$y(k) = \varphi^T(k)\,\theta + v(k) \ . \tag{17}$$

Here, let us use the notation $\hat{y}(k|\theta)$ as the one-step-ahead prediction value, then it is given by the linear formula with respect to the parameter vector θ as follows:

$$\hat{y}(k|\theta) = [1 - A(q^{-1})]y(k) + B(q^{-1})u(k) = \varphi^T(k)\,\theta \ . \tag{18}$$

The estimate model output $y_m(k)$ is constructed according to

$$y_m(k) = \varphi^T(k)\,\hat{\theta}(k) \ , \tag{19}$$

where the estimate vector of the parameter is defined as follows:

$$\hat{\theta}^T(k) = \{a_1(k), a_2(k), \cdots, a_n(k), b_0(k), b_1(k), \cdots, b_m(k)\} \ . \tag{20}$$

In conclusion, if the signal to noise (S/N) ratio of ARX system is a good condition, least-squares estimation has few bias with respect to the estimated value. A generalized adaptive identification based on a reliable least-squares estimation is described by means of the following formulation:

parameter adjusting:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \lambda_2(k)P(k)\varphi(k)\varepsilon(k) \tag{21}$$

adaptive gain:

$$P(k) = \frac{1}{\lambda_1(k)}\left\{P(k-1) - \frac{\lambda_2(k)P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda_1(k) + \lambda_2(k)\varphi^T(k)P(k-1)\varphi(k)}\right\} , \tag{22}$$

apriori error:

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \tag{23}$$

where weighting sequences $\lambda_1(k)$ and $\lambda_2(k)$ in (21) and (22) are $0 < \lambda_1(k) \le 1$ and $0 \le \lambda_2(k) < 2$, respectively.

Ⅱ-3 Characteristics Based On Optimal Servo

A discussion progressed by applying a binomial coefficient polynomial to simplify the tuning algorithm in [4], as the characteristic polynomial that is specified with (7). Type-1 optimal servo based on the state-space approach is tried in this paper. After the parameter estimation, a state-space description, for example, a controllable canonical form with respect to ARX system (1) is given by ignoring the disturbance noise $w(k)$ as $\Sigma : (\overline{A}, \overline{B}, \overline{C})$. Type-1 servo system that considered the computation time delay to the system Σ as shown in Fig.1 is constructed.

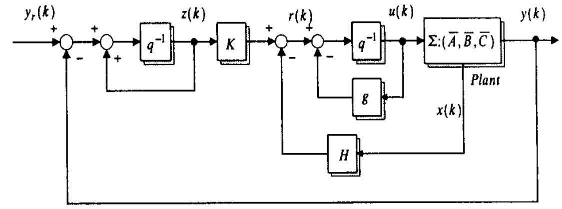

Fig.1. Type-1 optimal servo system having one sample delay.

Here, the quadratic type performance index is defined as

$$J = \sum_{k=0}^{\infty}\{\tilde{x}^T(k)Q\tilde{x}(k) + R\tilde{v}^2(k)\} \ , \quad R > 0 \ , \tag{24}$$

where $\tilde{x}(k)$ and $\tilde{v}(k)$ represent the states and actuating value for the extended deviation system, respectively, and Q is a semi-positive definite matrix. A type-1 optimal servo having one sample controller delay that minimizes a performance index of (24) is given as follows:

***Riccati* equation:**

$$P = Q + \overline{A}^T P \overline{A} - \overline{A}^T P \overline{B}(R + \overline{B}^T P \overline{B})^{-1}\overline{B}^T P \overline{A}, \quad P > 0 \tag{25}$$

Optimal feed-back gain:

$$F = (R + \overline{B}^T P \overline{B})^{-1}\overline{B}^T P \overline{A} \tag{26}$$

Controller parameters:

$$g = F\,\overline{B} + 1 \tag{27}$$

$$[H, K] = [F\overline{A}^2, F\overline{A}\overline{B} + F\overline{B} + 1]E^{-1} \tag{28}$$

$$E = \begin{bmatrix} \overline{A} - I & \overline{B} \\ \overline{C} & 0 \end{bmatrix} \tag{29}$$

Furthermore, the state-space description of this servo system is expressed as follows:

$$\begin{bmatrix} x(k+1) \\ u(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} & 0 \\ -H & -g & K \\ -\overline{C} & 0 & 1 \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k) \ . \tag{30}$$

Ⅱ-4 Intelligent Auto-tuning

First, The stability index is introduced in order to evaluate the relative stability of the control system. In the following n^{th} -order characteristic polynomial of the continuous time system:

$$p(s) = f_n s^n + \cdots + f_1 s + f_0 \ , \quad n \ge 2 \ , \tag{31}$$

the stability indices γ_i are generally defined as follows:

$$\gamma_i = f_i^2 / f_{i+1} \cdot f_{i-1} \ , \quad (i = 1, \cdots, n-1) \ . \tag{32}$$

A series compensator $[S(q^{-1})]^{-1}$ is derived by solving (14), according to the characteristic polynomial $D_o(q^{-1})$ that is obtained with an optimal design of the previous section.

Next, in order to apply a stability indices γ_i to the discrete-time series compensator $[S(q^{-1})]^{-1}$, it is transformed into the continuous-time transfer function $S_c(s)$ using the inverse bilinear transformation

$$q = (2 + Ts)/(2 - Ts) \ , \tag{33}$$

where T is the sampling period.

After the bilinear transformation, the stability indices γ_i in terms of the denominator polynomial of the continuous-time compensator $S_c(s)$ are calculated. Furthermore, such new index that evaluates relation between the stability of a controller and the performance of a closed-loop system is able to be composed of these stability indices γ_i . In the present paper, only the simplest resultant stability index γ is selected and is given by the algebraic product of the stability indices γ_i :

$$\gamma = \prod_{i=1}^{n-1} \gamma_i \ . \tag{34}$$

This new index γ can be related to the weight R of a performance index in the optimal control. Furthermore, the index γ is

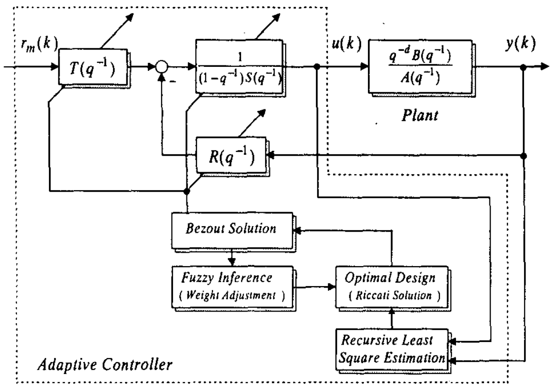

Fig.2. Block diagram of proposed adaptive control system.

related to each characteristic, whether open-loop, such as gain-phase margin, or close-loop, such as settling time. Appropriate auto-tuning of weight R based on the resultant stability index γ is achieved, by considering the relations as mentioned above. At the same time, the resultant stability index γ is selected as the scaling factor, and the fuzzy inference is introduced to adjust the weight R of a performance index in the optimal control.

In addition, in order to complete the defuzzification of the inference result, the consequent of fuzzy inference is executed using the min-max-gravity method of *Mamdani*:

$$u_g(k-1) = \{\sum x_j \cdot \mu(x_j)\} / \sum \mu(x_j) \tag{35}$$

where x_j is the nonfuzzy value, $\mu(x_j)$ is the value of the membership function, and $u_g(k-1)$ is the inference result.

In order to place the resultant stability index γ of the series compensator $S_c(s)$ into the specified area, the weight R of a performance index in the optimal control system is adjusted by means of the following recursive formula:

$$R(k) = R(k-1) \cdot 10^{\beta(k-1)} \tag{36}$$

$$\beta(k-1) = c \cdot u_g(k-1), \quad c : any\ constant\ . \tag{37}$$

These details are shown in a design example in the following section. Furthermore, the block diagram of the proposed adaptive control system is shown in Fig.2.

II -5 Design Example

A second order continuous-time plant is chosen as:

$$G(s) = \frac{K\omega_n^{\ 2}}{s^2 + 2\varsigma\omega_n s + \omega_n^{\ 2}}, \tag{38}$$

where K is assigned to a constant value of 1 in order to simplify the design. The controllers for both of the plant pairs, $\Sigma_f(\varsigma = 0.15, \omega_n = 55)$ and $\Sigma_s(\varsigma = 0.5, \omega_n = 15)$, are designed. First the plant Σ_f described above is transformed into a discrete-time system for sampling time $T = 0.02$[sec], because the digital controller is designed using a discrete-time model.

For example, a type-1 optimal servo controller having one sample controller delay [6] is calculated to the controllable canonical form under the condition that the weights of performance index of (24) are $Q = diag$ (100,100) and R =500.

Next, when *Riccati* equation of (25) is first solved, the optimal feed-back gain of (26) to a positive definite solution is given as

$$F = [\ -2.9628 \times 10^{-1} \quad 2.2826 \times 10^{-1}\ ], \tag{39}$$

the controller parameters are then obtained, respectively, as

$$g = 1.2283, H = [\ -0.47623 \quad 1.1118\ ], K = 9.2693 \times 10^{-1}. \tag{40}$$

At the same time, by using *Faddeev*'s algorithm, the characteristic polynomial $D_o(q^{-1})$ of (30) is calculated as follows:

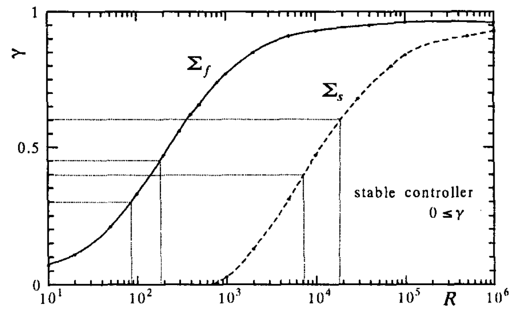

Fig.3. Each index γ of plants Σ_f and Σ_s to the weight R.

$$D_o(q^{-1}) = 1 - 5.5970 \times 10^{-1} q^{-1} + 4.2265 \times 10^{-1} q^{-2} \tag{41}$$

When (14) is solved after (41) is substituted to it, the pole placement controllers $S(q^{-1})$ and $R(q^{-1})$ are derived, respectively, as

$$S(q^{-1}) = 1 + 1.2283 q^{-1} + 5.9043 \times 10^{-1} q^{-2} \tag{42}$$

$$R(q^{-1}) = 1.0595 - 1.0998 q^{-1} + 0.96722 q^{-2}. \tag{43}$$

Furthermore, the scaling factor are obtained as $\gamma = 0.6573$, and also in order to confirm the absolute stability of $S_c(s)$, *Hurwitz* determinant is calculated as $H_1 = 81.915$. At this time, when the frequency response is calculated from the loop transfer function of this discrete-time system, the gain margin g_m and the phase margin p_m are calculated to be

$$g_m = 6.39 \quad [dB] \quad , \quad p_m = 65.0 \quad [\text{deg}] \tag{44}$$

respectively. In addition, settling time is confirmed as ST =12 [sample] through a step response simulation of only closed-loop system. Similarly, when the calculation is repeated changing R continuously, using fixed Q in terms of both weights, Q and R, in a performance index, each significant characteristic is obtained. The index γ for the performance weight R of both plants, Σ_f and Σ_s, is shown in Fig.3. However, other diagrams, that is, *Hurwitz* determinant, settling time etc. are omitted.

Next, the relation of both performance weight R and stability index γ having good performance for the control system of Σ_f and Σ_s is examined by considering each characteristics, for example, settling time etc. and Fig. 3. At this time, designer is able to choose the range flexibly as a suitable area of both stability and settling time. Consequently, one of the appropriate ranges for the purpose of auto-tuning is selected as follows:

$$\left.\begin{array}{lll} l_f \le \gamma_f \le u_f & , \ l_f = 0.3 & , \ u_f = 0.45 \\ l_s \le \gamma_s \le u_s & , \ l_s = 0.4 & , \ u_s = 0.6 \end{array}\right\}. \tag{45}$$

Fuzzy inference is then used to ensure a resultant stability index γ of the series compensator $[S(q^{-1})]^{-1}$ within the specified area.

The fuzzy variable γ used in the antecedent of the fuzzy rule has a trapezoidal membership function, as shown in Fig.4. In addition, the membership function used in the consequent of the fuzzy rule has a triangular form, as shown in Fig.5. The horizontal scale in the membership function of Fig. 4 can be selected as

$$JFI = \min(\gamma_f \cup \gamma_s), \quad SMS = \min(\gamma_f \cap \gamma_s)$$

$$LGI = \max(\gamma_f \cap \gamma_s)\ , \quad JFS = \max(\gamma_f \cup \gamma_s)\ . \tag{46}$$

The complete inference is found by calculating the center of gravity of (35) in terms of membership function shown in Fig. 5.

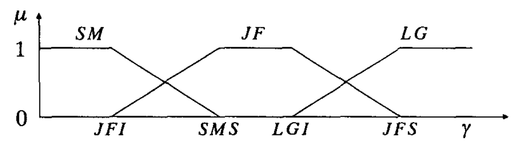

Fig.4. Membership function of antecedent.

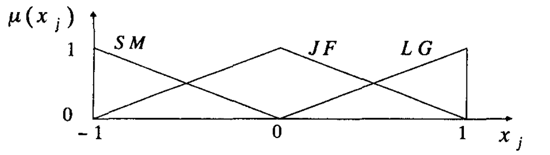

Fig.5. Membership function of consequent.

Ⅲ. SIMULATION RESULTS

The simulation result of the conventional adaptive pole placement control having the closed-loop pole designed by an optimal servo is shown. After that the auto-tuning simulation of adaptive control system is shown. It is assumed that the prefilter (8a) is specified and also the reference signal of $r_m(k) = u_m(k)$ without reference model is used for the plant of (38). When the weights of performance index (24) are chosen to be constant values of $Q = diag$ (100,100)and R =500, the simulation result of the adaptive pole placement control by the conventional method is first shown in Figs. 6 and 7. The plant parameter has been changed to Σ_s from Σ_f at a step of 400 and a minimal disturbance noise of variance $\mathrm{var} = 5\times10^{-6}$ has been added to the simulation. The control result appears to be good according to Fig. 6; however, the series compensator is unstable in the area of the Σ_s plant. When an unstable compensator appears in the pole placement system, Fig.7 reveals that the manipulated variable is greatly disturbed even for a minimal random disturbance. Consequently, a strongly stable system is not achieved using the conventional method even if optimal servo system is designed to the given plant, and the selection of an appropriate closed-loop pole becomes difficult.

The simulation result obtained by using the proposed method in same condition mentioned above is next shown in Figs.8 through 10. The controlled variable $y(k)$ of the plant is shown in Fig.8, and the performance weight R in optimal servo and the resultant index γ are shown in Figs. 9 and 10, respectively. That the performance weight R is adjusted appropriately and simultaneously the occurrence of unstable controller is prevented in accordance with the proposed method, even after the plant parameters are changed greatly at a step of 400 in Fig.9 is obvious. Furthermore, Fig.10 reveals that the proposed adaptive control system is appropriately adjusted by means of fuzzy inference as the stability index γ is put in the specified area.

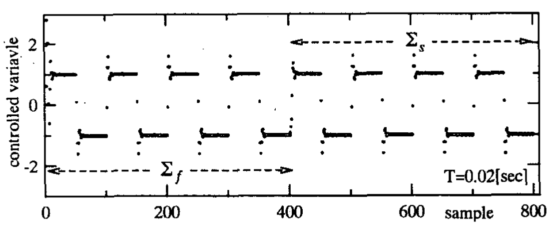

Fig.6. Index γ , in case of conventional adaptive control.

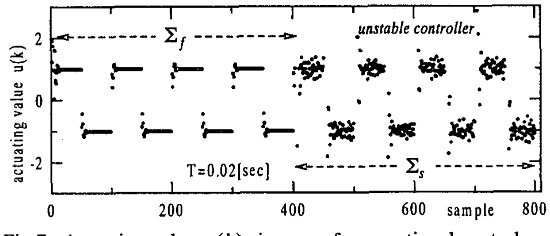

Fig.7. Actuating value $u(k)$, in case of conventional control.

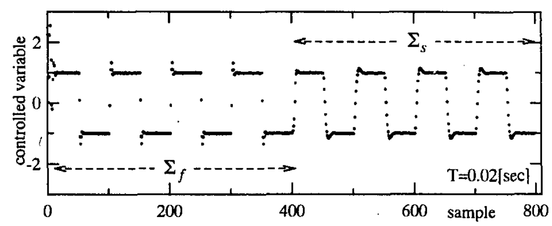

Fig.8. Controlled variable $y(k)$, in case of proposed method.

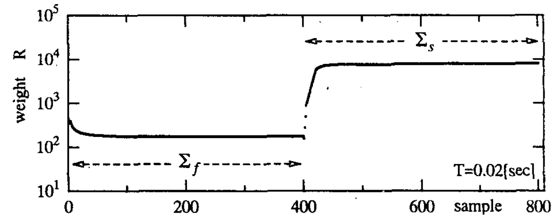

Fig.9. Performance weight R, in case of proposed method.

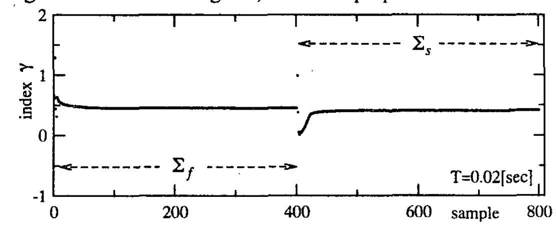

Fig.10. Index γ , in case of proposed method.

Ⅳ. CONCLUSIONS

This paper has proposed a useful design strategy for automatically constructing the strongly stable adaptive control system. An original and evident solution for the control purpose is derived like [4] by applying the proposed scheme in terms of this problem; however, the unstable series compensator is guided into the control system on the occasion of the use of conventional method. The proposed method has been applied to some systems similar to [4] as a design example, and the usefulness of a strategy was demonstrated. It was able to inspect the real time performance regarding the proposed algorithm through some simulation too. In other words, the proposed method was able to confirm even that is sufficiently applicable to practical machine.

REFERENCES

[1] K.J.Åström & B.Wittenmark, *Adaptive Control*, Addison Wesley, 1989.

[2] I.D.Landau, *System Identification and Control Design*, Prentice Hall, 1990.

[3] J.C.Doyle, B.A.Francis & A.R.Tannenbaum, *Feedback Control Theory*, Macmillan, 1992.

[4] P.Ratiroch-anant, W.Surakampontorn, H.Hirata, M.Anabuki & J.Ngamwiwit, *"Fuzzy Adaptive Pole Placement Control Considering Stability Index"*, IEEE Asia-Pacific Conference on Circuits and Systems, Chiangmai, 1998, pp.667-670.

[5] S.Manabe, *"Coefficient Diagram Method as Applied to the Attitude Control of Controlled-Bias-Momentum Satellite "*, 13th IFAC symposium on automatic control in aerospace, Palo Alto, CA., 1994, pp.322-327.

[6] T.Mita, *"Optimal Digital Feedback Control Systems Counting Computation Time of Control Laws "*, IEEE Trans. on Automatic Control, Vol30, No6, 1985, pp.542-548.

[7] T.Terano, K.Asai & M.Sugeno, *Fuzzy Systems Theory and Its Applications*, Academic Press, 1992.