Fuzzy Adaptive Pole Placement Control Considering Stability Index

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Abstract

This paper proposes a method for automatically adjusting the pole of a stable closed-loop system and presents a design strategy for a strongly stable adaptive system. When the system parameter fluctuates widely, a controller having unstable poles appears in an adaptive pole placement control system, even if the closed-loop system is selected to be a stable system. By examining the stability indices of the series compensator, the designer can easily obtain a unique solution to the unstable controller. A design method for an adaptive system that recursively adjusts the poles of a stable closed-loop system using fuzzy inference is proposed.

1. Introduction

Controlled system parameters usually fluctuate over a wide range, for example, the inertia fluctuation of a motor very often appears in the speed control on the rolling lines of iron mills. Moreover, control performance usually deteriorates when applying conventional control methods. Therefore, adaptive control [1], [2], is a very effective method for such system. The present study used the indirect method of adaptive pole placement control [1] and introduced a series compensator coupled with the integrator in order to reject constant disturbances.

When the parameters fluctuate over a wide range, a series compensator having an unstable pole often appears in the adaptive pole placement control system, even if the closed-loop was designed as a stable system. The appearance of the unstable compensator is not desirable with respect to stability. Therefore, the unstable controller is reluctantly used, especially if the plant itself is a stable system. In order to obtain a stable controller over the full range of fluctuation, appropriate selection of the pole of a closed-loop system is required.

However, selecting the desired closed-loop pole under wide range of parameter fluctuation conditions is difficult. Few design methods consider the relative stability of the pole placement system itself; however, several methods consider the robustness [3] of the adaptive law. Furthermore, adjustment of both the controller and the closed-loop poles according to the fluctuation of the plant parameter has not yet been reported. Recently, we announced an effective method [4] that constructs a strongly stable system [5], even if the plant parameter fluctuates widely. In this paper, a unique and strongly stable adaptive control system, which recursively adjusts the series compensator using both adaptive identification and fuzzy inference [6], by examining the stability indices [7], [8], is verified.

2. Fuzzy adaptive pole placement control

2.1 Identification system

The block diagram of the proposed adaptive control system is shown in Fig.1. The plant is described by the ARX (auto-regressive exogenous) model

$$y(k)=\frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k)+\frac{1}{A(q^{-1})}v(k)\ , \qquad (1)$$

where $v(k)$ is the white noise having zero mean value, and q^{-1} is the delay operator , the plant denominator

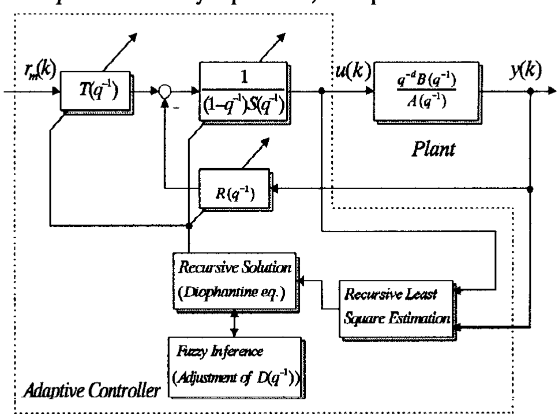

Fig.1. Block diagram of proposed adaptive control system.

$A(q^{-1})$ and numerator $B(q^{-1})$ are given as follows:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} , \quad (2)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} . \quad (3)$$

If the parameter vector θ and the regression vector $\varphi(k)$ are defined as

$$\theta^T = \{ a_1 , a_2 ,\cdots, a_n ,b_0 ,b_1 ,\cdots, b_m \} \quad (4)$$

and

$$\varphi^T(k) = \{ -y(k-1), -y(k-2), \cdots, -y(k-n), u(k-d), u(k-1-d), \cdots, u(k-m-d) \} , \quad (5)$$

respectively, then the plant output $y(k)$ is expressed as

$$y(k) = \varphi^T(k)\ \theta + v(k) . \quad (6)$$

In addition, let us use the notation $\hat{y}(k|\theta)$ as the one-step-ahead prediction value, which can be described by a linear equation with respect to the parameter vector θ as

$$\hat{y}(k|\theta) = [1 - A(q^{-1})]y(k) + B(q^{-1})u(k) = \varphi^T(k)\theta . \quad (7)$$

The estimate output $y_m(k)$ will be selected according to

$$y_m(k) = \varphi^T(k)\ \hat{\theta}(k) , \quad (8)$$

where the estimate vector $\hat{\theta}(k)$ of the parameters is defined as follows:

$$\hat{\theta}^T(k) = \{a_1(k), a_2(k), \cdots, a_n(k),\ b_0(k), b_1(k), \cdots, b_m(k)\} . \quad (9)$$

In conclusion, an RLS (recursive least-squares) algorithm suitable for adaptive control can be obtained. And the generalized estimate algorithm based on the RLS method is given by the following equations:

parameter adjusting:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{1 + \varphi^T(k)P(k-1)\varphi(k)}\ \varepsilon(k) \quad (10)$$

apriori error:

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \quad (11)$$

adaptive gain:

$$P(k) = \frac{1}{\lambda_1(k)}\left\{ P(k-1) - \frac{\lambda_2(k)P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda_1(k) + \lambda_2(k)\varphi^T(k)P(k-1)\varphi(k)} \right\} \quad (12)$$

where, in this present study, the weighting sequences $\lambda_1(k)$, $\lambda_2(k)$ of the adaptive gain are $0 < \lambda_1(k) \le 1$ and $0 \le \lambda_2(k) < 2$. However, it should be noted from (12) that, for the other designer, the gain can be obtained by selecting the appropriate values of $\lambda_1(k)$ and $\lambda_2(k)$.

2.2 Pole placement control system

The pole placement controller, in general, is designed to be adaptable to unknown plant parameters, where the parameters are estimated by the identification system in section 2.1. In order to reject stationary disturbances, an integrator is introduced to the control system. In this way, the designer is able to regard the pole placement control as a special case of GMV (generalized minimum variance control) [1],[2]. In other words, the controller is easily formulated by ignoring the disturbance noise $v(k)$ of (1). If the poles of the stable closed-loop system are specified by the $n_d{}^{th}$ -order polynomial

$$D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d} ,\quad n_d \le n + m + d , \quad (13)$$

then, the output of the controlled plant is given by

$$D(q^{-1})y(k) = q^{-d}T(q^{-1})B(q^{-1})r_m(k) , \quad (14)$$

where $r_m(k)$ is the reference input signal and $T(q^{-1})$ is the feed-forward precompensator. In general, either the constant gain $T(q^{-1}) = D(1)/B(1)$ or the polynomial $T(q^{-1}) = D(q^{-1})/B(1)$ is used as the precompensator.

Also, the manipulated variable $u(k)$ of the plant is designed by means of the following signal synthesis:

$$(1 - q^{-1})S(q^{-1})u(k) = T(q^{-1})r_m(k) - R(q^{-1})y(k) , \quad (15)$$

where $S(q^{-1})$ is the forward controller (or the series compensator) and $R(q^{-1})$ is the feedback controller. Furthermore, the controllers $S(q^{-1})$ and $R(q^{-1})$ are given by the following polynomials:

$$S(q^{-1}) = 1 + s_1 q^{-1} + \cdots + s_{n_s} q^{-n_s} ,\quad n_s = m + d - 1, \quad (16)$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \cdots + r_{n_r} q^{-n_r} ,\quad n_r = n . \quad (17)$$

When $u(k)$ of (15) is substituted into (1), the following equation is obtained:

$$[(1 - q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})]y(k) = q^{-d}T(q^{-1})B(q^{-1})r_m(k) . \quad (18)$$

Furthermore, comparing (14) and (18) reveals that obtaining the desired response requires the solution of

$$D(q^{-1}) = (1 - q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) . \quad (19)$$

This equation is generally refered to as the *Diophantine* equation (or the *Bezout* identity) [1],[2].

2.3 Fuzzy adaptive pole placement control based on stability index [7],[8]

When the poles of stable closed-loop system to the unknown plant are arbitrarily selected, the unstable compensator often appears in the pole placement system. It occurs more frequently if the plant parameters widely fluctuate. In order to evaluate the stability of the control system, the stability index is introduced. In the following characteristic polynomial $p(s)$ of the continuous system transfer function:

$$p(s) = f_n s^n + \cdots + f_1 s + f_0 ,\quad n \ge 2 , \quad (20)$$

the stability indices γ_i is generally defined as follows:

$$\gamma_i = f_i^2 / f_{i+1} \cdot f_{i-1} ,\quad (i = 1, \cdots, n-1) . \quad (21)$$

Next, the discrete compensator $[S(q^{-1})]^{-1}$ is transformed into the continuous transfer function $S_c(s)$ using the bilinear inverse transformation

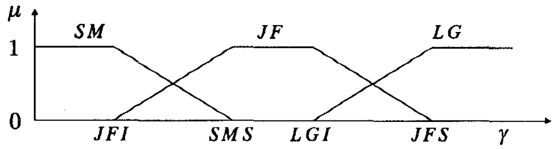

Fig.2. Membership function of antecedent.

$$q = (2 + Ts)/(2 - Ts), \quad T: \textit{sampling period}. \quad (22)$$

After that, the stability indices γ_i of the denominator polynomial in terms of continuous transfer function $S_c(s)$ are calculated. For easy discussion one simple index γ is constructed from the stability indices γ_i as follows:

$$\gamma = \prod_{i=1}^{n-1} \gamma_i \quad (23)$$

where this new index γ is related to the open-loop characteristics such as gain-phase margin, etc. And as the control specification is always achieved, the stability index γ of the series compensator $[S(q^{-1})]^{-1}$ is placed into the specific area by using the fuzzy inference.

Namely this index γ is named as a fuzzy variable and the membership function used in the antecedent of the fuzzy rule has the trapezoidal shape as shown in Fig.2. And also the membership function used in the consequent of the fuzzy rule has the triangular form as shown in Fig.3. Furthermore, in order to complete the defuzzification of the inference result, the following min-max-gravity method is used:

$$u_g(k-1) = \{\Sigma x_j \cdot \mu(x_j)\}/\Sigma \mu(x_j). \quad (24)$$

In order to place the stability index γ of the compensator $S_c(s)$ into the specific area, the pole α of the closed-loop system is adjusted using the following formula

$$\alpha(k) = \alpha(k-1) + c \cdot u_g(k-1), \quad c: \textit{constant} \quad (25)$$

3. Design example

The 3^{rd} order continuous plant is selected as:

$$G(s) = \frac{K\omega_n^{\ 2}}{(s+d)(s^2 + 2\varsigma\omega_n s + \omega_n^{\ 2})} \quad (26)$$

where K and d are assigned a constant value of 50 for simplicity. Now the digital controllers for both following plants $\Sigma_f (\varsigma = 0.1, \omega_n = 60)$ and $\Sigma_s (\varsigma = 0.4, \omega_n = 20)$ are designed and the sampling period is selected as $T = 0.02$ [sec]. Then, the stability index γ, the gain margin g_m, the phase margin p_m and the gain-crossover frequency ω_{cg} to the closed-loop pole α are calculated, respectively. Similarly, when the above calculation is repeated changing the closed-loop pole α continuously, four most important characteristics can be obtained, respectively. The index γ to the closed-loop pole α of the parameter pairs Σ_f and Σ_s is shown in Fig.4.

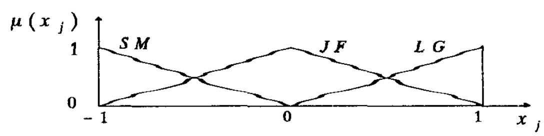

Fig.3. Membership function of consequent.

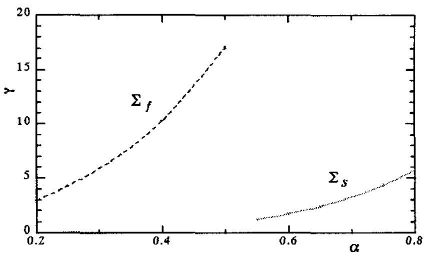

Fig.4. Index γ of plants Σ_f and Σ_s to closed-loop pole α.

If Table 1 is given as the control specification, then the range of the closed-loop pole that fills this specification is obtained like the following form from the above-obtained characteristics:

$$0.2 \le \alpha_f \le 0.32, \quad 0.62 \le \alpha_s \le 0.75, \quad (27)$$

where the closed-loop poles α_f and α_s represent the range of plants Σ_f and Σ_s, respectively. Therefore, if Inequality (27) that fills the control specification is considered, then the range of the index γ which has good performance for the control system can be decided from Fig.4 as follows:

$$\left.\begin{aligned} l_f \le \gamma_f \le u_f, \quad l_f = 3, \quad u_f = 6.6 \\ l_s \le \gamma_s \le u_s, \quad l_s = 2, \quad u_s = 4.2 \end{aligned}\right\} \quad (28)$$

where the indices γ_f and γ_s represent the range of plants Σ_f and Σ_s, respectively. The horizontal scale in membership function of Fig.2 can be usually selected as the common part $\gamma_f \cap \gamma_s$ of Inequality (28). In order to reduce the range of SMS and LGI in this example, the next relation can be selected for membership function:

$$JFI = \min(l_f, l_s), \quad SMS = (u_s + l_f)/2,$$
$$LGI = (u_f + l_s)/2, \quad JFS = \max(u_f, u_s). \quad (29)$$

4. Simulation result

First, the adaptive pole placement control by the conventional method is simulated and the stable closed-loop pole α is chosen to be a constant value of 0.45 for the plant of (26). The plant parameter has been changed to $\Sigma_{s1} (\varsigma = 0.35, \omega_n = 25)$ from $\Sigma_{f1} (\varsigma = 0.15, \omega_n = 55)$

Table 1 Control specification

	g_m [dB]	p_m [deg]	ω_{cg} [rad / sec]
Σ_f	$g_m \ge 6.5$	$p_m \ge 60$	$\omega_{cg} \ge 9$
Σ_s	$g_m \ge 6.5$	$p_m \ge 60$	$\omega_{cg} \ge 4$

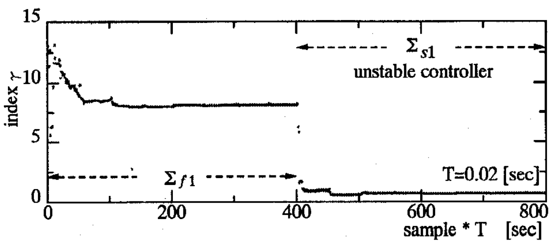

Fig.5. Index γ, in case of conventional adaptive pole placement method (with constant value of $\alpha = 0.45$).

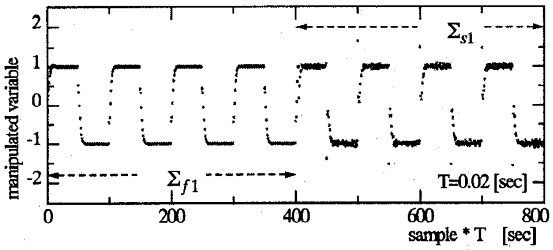

Fig.6. Manipulated variable synthesized by conventional adaptive pole placement (with $\alpha = 0.45$).

at a step of 400 and very minimal disturbance noise of variance $\sigma_n^2 = 5 \times 10^{-6}$ has been added to the simulation. The stability index γ and the manipulated variable $u(k)$ are shown in Figs. 5 and 6, respectively. It is confirmed in Fig.5 that the index γ is very small in the area of the parameter Σ_{s1} and also the compensator is unstable. Therefore, Fig.6 reveals that the manipulated variable is greatly disturbed even for a minimal random disturbance. Consequently, a strongly stable system is not realized using the conventional method.

The simulation result obtained using the proposed method is shown in Figs. 7 through 9. The plant output $y(k)$ is shown in Fig.7 and the stability index γ and the closed-loop pole α are shown in Figs. 8 and 9, respectively. The plant parameter is changed to Σ_{s1} from Σ_{f1} at a step of 400. As the stability index γ is placed in the designated range, the closed-loop pole α is automatically adjusted in Figs. 8 and 9. Therefore, a strongly stable system is easily realized using the proposed method, even if the plant parameters fluctuate greatly, as in the present simulation.

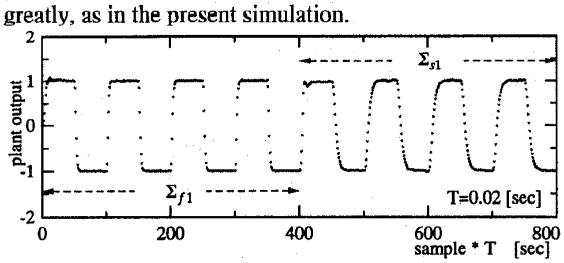

Fig.7. Plant output $y(k)$ controlled by proposed method.

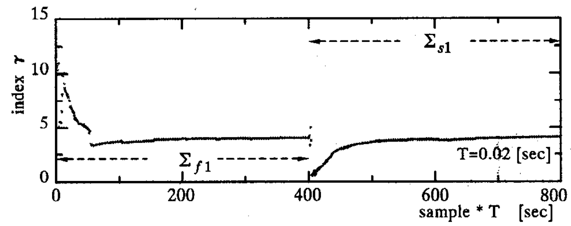

Fig.8. Index γ, in case of proposed method.

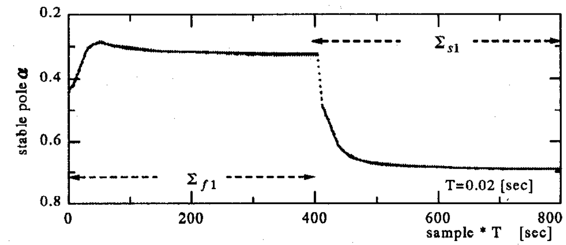

Fig.9. Closed-loop pole α, in case of proposed method.

5. Conclusions

In the present paper, the design method of a strongly stable system for the adaptive pole placement control has been proposed. A unique solution was obtained for this problem using the proposed scheme; however, the unstable compensator is guided by the control system in the conventional method. The proposed method was applied to a 3^{rd}-order system as an application example, and the usefulness of this method was demonstrated.

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