# Auto-Tuning Adaptive Control System for DC Motor Speed Control

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## Abstract

The auto-tuning method for the strongly stable adaptive control of DC motor speed system with great variation of the load inertia is proposed. The stable closed-loop characteristic polynomial that is designed by type-1 optimal servo with one sample delay is specified for the adaptive pole placement control. The appropriate adaptive control system can be derived, by adjusting automatically the weight of performance criterion in the optimal servo by means of fuzzy inference on the basis of the stability index. Furthermore, the transient characteristic is improved by tuning the tracking model. The experiment results confirm the control performance of the proposed strategy.

## 1. Introduction

The DC motor speed control system with fixed controller is often used in the industries. When the load inertia of DC motor greatly changes, usually the control performance deteriorates. Adaptive control [1], [2] is very effective well known strategy for such control system. However, when the load inertia changes over a wide range. The unstable poles frequently appear in the compensator of adaptive pole placement system, even if the closed-loop characteristic has been designed as a stable system or the plant itself is a stable system. This is not desirable with respect to both stability and reliability. Therefore, if the designer is able to adjust the stable poles of a closed-loop system recursively according to the load inertia variation, the construction of a strongly stable system [3] is easily realized. Recently, the authors announced some effective design methods [4], [5] for constructs a strongly stable adaptive pole placement system even if the plant parameter greatly changes. This paper is application result of the previous research.

An optimal servo system [6] is recursively designed by estimating the DC motor speed-control system. The series controller is automatically constructed by solving the *Bezout* identity on the basis of the derived optimal servo characteristic polynomial. After the stability index of the series compensator is examined, the weight in an optimal control design is automatically updated by using the fuzzy inference [7]. That is, the adaptive system can places the stability index of the series controller into the specified region according to the adjusting weight.. This method ensures the stability of both the closed-loop system and the controller, it also can achieves fine control performance.

## 2. Driver amplifier and model of DC motor

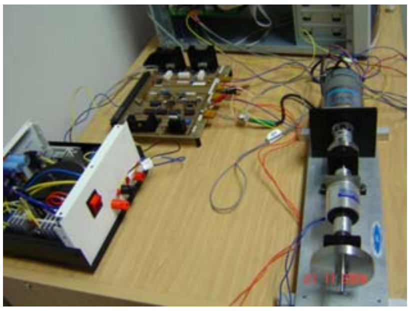

**Fig 1** The DC Motor set for performing the speed control.

The DC motor set used in the experiment and its block diagram of DC motor driver are shown in Figure 1 and Figure 2, respectively. The steel disk is the light load of this motor (24V-20W) and the heavy load is performed by an electric clutch (DC-24V) at the motor shaft. The DC motor is driven by the servo amplifier based on type-1 optimum servo design and its speed is measured by an optical encoder (1000P/R). The equivalent disturbance control and the reduction of uncertainty are realized by using this servo driver, and the original performance of adaptive control is skillfully achieved. The symbols used in the design of DC motor driver are defined as follows:

$L$ , $R_a$ : armature inductance $[H]$ and resistance $[\Omega]$
$i$ : armature current $[A]$ , $K_p$ : power amplifier gain
$K_\tau$ : torque constant $[Nm/A]$
$K_e$ : back electromotive force constant $[V\sec/rad]$
$J_m$ : momentum of rotor inertia $[kgm^2]$
$J_\ell$ : momentum of load inertia $[kgm^2]$
$\omega$ : rotor speed $[rad/\sec]$ , $\tau_f$ : disturbance torque $[Nm]$
$\omega_V$ : voltage output of rotor speed $[V]$
$S_V$ : conversion constant $\omega$ to $\omega_V$ $[V\sec/rad]$
$r$ : applied voltage of power amplifier $[V]$
$R_i$ : resistance for armature current detection $[\Omega]$

The state-space description of DC motor is expressed by disregarding a disturbance torque as follows:

$$\begin{cases} \dot{x} = A\,x + B\,r \\ y = C\,x \end{cases}, \quad x = [\omega \quad i]^T, \quad y = \omega_V , \tag{1}$$

where $A$, $B$ and $C$ are defined as

$$A = \begin{bmatrix} 0 & K_\tau/J \\ -K_e/L & -R_t/L \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ K_p/L \end{bmatrix}, \quad C = [S_V \quad 0],$$
$$J = J_m + J_\ell , \quad R_t = R_a + R_i . \tag{2}$$

The extended system introduced with an integrator is described as

$$\begin{bmatrix}\dot{x}\\ \dot{z}_1\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x\\ z_1\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}r\,, \tag{3}$$

and actuating value is constructed by the following control:

$$r=-F\,x+K_1\,z_1\ ,\quad F=[f_1\quad f_2]\,, \tag{4}$$

where $f_1$, $f_2$ and $K_1$ are feedback gains, respectively.

In general, the feedback gains can be decided by the pole placement method or the optimal servo design. The procedure of type-1 optimal servo design is summarized as follows. The extended deviation system of DC motor described as (1) is given by following equations:

$$\begin{cases}\dot{X}=A_1\ X+B_1\ v\\ e=C_1\ X\end{cases},\quad X=\begin{bmatrix}x-x_S\\ r-r_S\end{bmatrix},\quad e=y-u_S\,, \tag{5}$$

where the variables $x_S$, $r_S$ and $u_S$ represent the steady state of the variables $x$, $r$ and $u$, respectively, and $A_1$, $B_1$ and $C_1$ are defined as

$$A_1=\begin{bmatrix}A & B\\ 0 & 0\end{bmatrix},\quad B_1=\begin{bmatrix}0\\ 1\end{bmatrix},\quad C_1=[C\quad 0]\ . \tag{6}$$

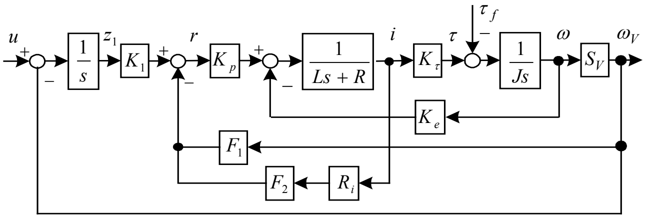


**Fig. 2** The block diagram of DC motor driver

At this time, the optimal servo problem is considered an optimal regulator problem, which operates the following feedback control to the system of (5):

$$v=-F_X X\,,\quad F_X=[F\quad K_1]\begin{bmatrix}A & B\\ C & 0\end{bmatrix}. \tag{7}$$

Therefore, when the performance criterion is defined as

$$J=\int_0^\infty (X^T C_1{}^T W C_1{}^T X+v^T R_c\, v)dt\,,\quad W>0, R_c>0\ , \tag{8}$$

the optimal feedback $v^o$ that minimizes (8) is given by

$$v^o=-F_X{}^o X\,,\quad F_X{}^o=R_c{}^{-1}B_1{}^T P_c{}^o\,, \tag{9}$$

where $P_c^o$ is the solution of following *Riccati* equation:

$$A_1{}^T P_c+P_c A_1-P_c B_1 R_c{}^{-1} B_1{}^T P_c+C_1{}^T W C_1=0\,. \tag{10}$$

After the feedback gains in (9) were obtained, the practical feedback is performed by using the following optimal gain:

$$[F\ \ K_1]=F_X{}^o\begin{bmatrix}A & B\\ C & 0\end{bmatrix}^{-1},\quad F_1=f_1/S_V\ ,\quad F_2=f_2/R_i\,. \tag{11}$$

Furthermore, the closed-loop system is described as

$$\begin{bmatrix}\dot{x}\\ \dot{z}_1\end{bmatrix}=\begin{bmatrix}A-BF & BK_1\\ -C & 0\end{bmatrix}\begin{bmatrix}x\\ z_1\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u\,,\quad \omega_V=C\,x\,. \tag{12}$$

Consequently, the transfer function to the motor speed $\omega_V$ from the reference $u$ can be obtained as follow:

$$\frac{\omega_V}{u}=\frac{K}{JLs^3+J(R+K_p f_2)s^2+K_\tau(K_p f_1+K_e)s+K}\,, \tag{13}$$

where the constant gain $K$ is $K=K_1 K_p K_\tau S_V$.

## 3. Adaptive control system

The proposed adaptive control system shown in Fig.3 does not introduce the pole-zeros cancellation. The adaptive pole placement control is appropriately achieved by repeating both estimation and control alternately, and the structure of the system between the control and estimation are separated.

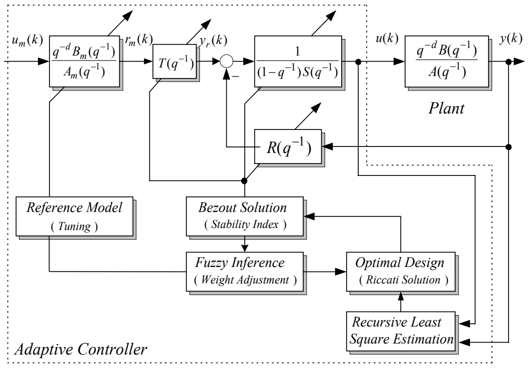


**Fig. 3** Block diagram of adaptive control system

The plant described by ARX model is controlled by RST–structure controller. When the plant parameters are unknown or greatly change, the estimated value is substituted in the control system. In this paper the identification technique base on Recursive Least Square method is used. The estimated DC motor system of the plant is described by state space equation .In order to minimize performance index, *Riccati* equation is applied.

$$P=Q+\overline{A}^T P\overline{A}-\overline{A}^T P\overline{B}(R+\overline{B}^T P\overline{B})^{-1}\overline{B}^T P\overline{A}\ , \tag{14}$$

The optimal feedback gain $F$ and control parameter $H$,$K$, which are obtained [5] from the solution of *Riccati* equation, are rewritten in extended system that leads to desired characteristic polynomial $D_0(q^{-1})$ Then $R(q^{-1})$, $S(q^{-1})$ and $T(q^{-1})$ can be derived by solving the *Bezout* identity.

$$D_o(q^{-1})=(1-q^{-1})A(q^{-1})S(q^{-1})+q^{-d}B(q^{-1})R(q^{-1})\,. \tag{15}$$

Base on relationship between stability index $\gamma$ [5] and weight $R$ of a performance index and use the fuzzy inference to adju st of this weight $R$ in the optimal control and also improves t he transient characteristic by tuning the pole of the reference model.

## 4. Control system design example

The major constants of DC motor are shown in Table 1. The servo driver amplifier of DC motor shown in Fig.2 is constructed by using an optimal servo design under the load

**Table 1** The design constants of DC motor

| DC motor 24V-20W | |
|---|---|
| $K_\tau=7.154\times10^{-2}[Nm/A]$ | $K_e=7.162\times10^{-2}[V\sec/rad]$ |
| $J_m=0.4\times10^{-4}[kgm^2]$ | $S_v=3.183\times10^{-2}[V/rad/\sec]$ |
| $R_a=4.3[\Omega]$, $R_i=0.2[\Omega]$ | $K_P=15$ , $L=6$ $[mH]$ |

inertia $J=1.5J_m$ ($J_\ell=0.2\times10^{-4}$) .When both performance weights $W$ and $R$ of (8) are selected as $W=60$ and

$R = 1/150$, respectively, the optimal feedback gains are obtained as $F_1 = 1.19$, $F_2 = 0.276$ and $K_1 = 94.9$.

The proposed method is verified by using the reduced order discrete-time model, because DC motor speed system in (13) is possible to reduce the system order by examining the singular value. The new method is easily designed by using discrete-time models of both plant $\Sigma_f (J_f \cong 1.5 J_m)$ and $\Sigma_s (J_s \cong 5 J_f)$ in order to adapt to great variation of the load inertia. When the sampling time $T$ is selected as $0.005[\mathrm{sec}]$, the discrete-time models of $\Sigma_f$ and $\Sigma_s$ which are estimated by the recursive least-squares method are shown respectively, as

$$\Sigma_f : G_f(q^{-1}) = \frac{q^{-1}(0.011543 + 0.17697 q^{-1})}{1 - 1.33901 q^{-1} + 0.52905 q^{-2}}, \quad (16)$$

$$\Sigma_s : G_s(q^{-1}) = \frac{q^{-1}(0.0045247 + 0.055808 q^{-1})}{1 - 1.77579 q^{-1} + 0.83703 q^{-2}}. \quad (17)$$

Due to the computation time delay, $d$ should be increased by 1, yielding $d = 2$. The closed-loop system of second order is derived by optimal servo design in the case of these motor design models. It has been confirmed by checking stability index that the sufficient stability could not be ensured in the case of this control characteristic. Therefore, appending the virtual pole-zero pair $P_V = 0.8$ to the both plant models, in order to design the closed-loop polynomial $D_o(q^{-1})$ of the third order. The concrete procedure of the design is shown as follows in the case of the plant $G_f(q^{-1})$. Type-1 optimal servo is calculated under a condition that the weights are $Q = diag\ (100,100,100)$ and $R = 1 \times 10^5$, respectively.

Then, the controller parameters are obtained, respectively, as

$$g = 1.116, \quad K = 1.664, \quad H = [0.2366 \ \ -1.319 \ \ 1.224]. \quad (18)$$

The characteristic polynomial is calculated as

$$D_o(q^{-1}) = 1 - 2.0232 q^{-1} + 1.4609 q^{-2} - 0.3749 q^{-3}. \quad (19)$$

$S(q^{-1})$ and $R(q^{-1})$ are given, respectively, as

$$S(q^{-1}) = 1 + 0.31577 q^{-1} + 0.31064 q^{-2} \quad (20)$$

$$R(q^{-1}) = 1.7998 - 2.3957 q^{-1} + 0.92867 q^{-2}. \quad (21)$$

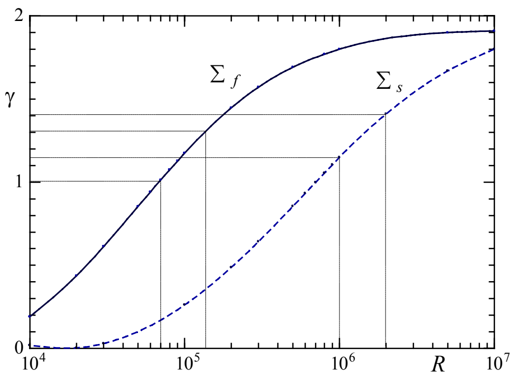


**Fig. 4** Each index $\gamma$ of plants $\Sigma_f$ and $\Sigma_s$ to weight $R$

Furthermore, the stability index is obtained as $\gamma = 1.175$, and also in order to confirm the absolute stability of $F_c(s)$, *Hurwitz* determinant is calculated as $H_1 = 551$. The gain margin $g_m$ and the phase margin $p_m$ are calculated from the loop transfer function of this discrete-time system as

$$g_m = 9.68\ [dB], \quad p_m = 69.2\ [\mathrm{deg}], \quad (22)$$

respectively. In addition, settling time is confirmed as $ST$ =11~14 [sample] through a step response experiment of only closed-loop system. Similarly, when the calculation is repeated changing $R$ continuously, using fixed Q in terms of both weights, Q and $R$, each significant characteristic is obtained. The index $\gamma$ to the weight $R$ of both plants, $\Sigma_f$ and $\Sigma_s$, are shown in Figs.4. In this paper other diagram, that is *Hurwitz* determinant, setting time etc. are omitted. Next, the relation of both weight $R$ and stability index $\gamma$ having good performance for the control system of $\Sigma_f$ and $\Sigma_s$ are examined by considering each characteristic, for example, setting time, Fig.4 and etc. Then, the designer is able to choose the flexible range as a suitable area of both stability index and setting time. Consequently, one of the appropriate ranges for the auto-tuning is selected as

$$\left. \begin{array}{ll} l_f \le \gamma_f \le u_f, & l_f = 1.0, \quad u_f = 1.3 \\ l_s \le \gamma_s \le u_s, & l_s = 1.15, \quad u_s = 1.4 \end{array} \right\}. \quad (23)$$

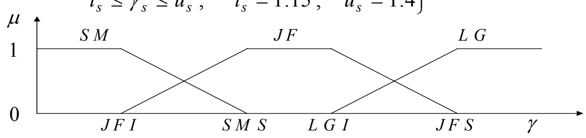


**Fig. 5** Membership function of antecedent

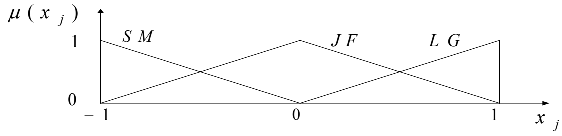


**Fig. 6** Membership function of consequent

Fuzzy inference is used to ensure the stability index $\gamma$ of compensator $[S(q^{-1})]^{-1}$ within a specified area. The fuzzy variable $\gamma$ used in the antecedent of the fuzzy rule has a trapezoidal membership function, as shown in Fig. 5. The membership function used in the consequent of the fuzzy rule has a triangular form, as shown in Fig.6. The horizontal scale in the membership function of Fig. 5 can be selected as

$$JFI = \min(\gamma_f \cup \gamma_s), \quad SMS = \min(\gamma_f \cap \gamma_s)$$

$$LGI = \max(\gamma_f \cap \gamma_s), \quad JFS = \max(\gamma_f \cup \gamma_s). \quad (24)$$

The complete inference is found by calculating the center of gravity in terms of membership function is shown in Fig. 6.

The transient characteristic can be improved by binomial coefficient tracking model as follow

$$A_m(q^{-1}) = (1 - \alpha q^{-1})^3, \quad B_m(q^{-1}) = A_m(1), \quad (25)$$

where $\alpha$ is the adjustable parameter of $\alpha < 0.8$.The relation of setting time and stable adjustable pole $\alpha$ is obtained through the step response simulation of tracking specification:

$$G_{tr} = \frac{q^{-2} A_m(1)}{A_m(q^{-1})} \cdot \frac{B(q^{-1})}{B(1)}. \quad (26)$$

By considering the characteristic of (26), it is confirmed that both plants $\Sigma_f$ and $\Sigma_s$ have similar settling time, even if the load inertia greatly changes. Therefore, the transient response can be improved by tuning the pole $\alpha$ of tracking model accordingly. The tuning formula is given by

$$\alpha(k) = a \log R(k) + b \ \ ; \ a = 0.207, \quad b = -0.603. \quad (27)$$

## 5. Experiment results

Two experiment results for the adaptive control of DC motor speed are shown. As condition, the performance weight $Q = diag\ (100,100)$ and the initial value of $R = 1\times10^5$. The load inertia of DC motor has been changed to $\Sigma_s$ from $\Sigma_f$ at a step of 400 and a disturbance noise of variance $5\times10^{-6}$ has been added to the experiment. First, the experiment results without the tracking model of (17) are shown in Figs. 8. The performance weight $R$ is adjusted appropriately and the required controller can automatically be obtained in each load inertia even after the load inertia are changed greatly at a step of 400 in Fig.9. However, an overshoot is observed in the transient response in Fig. 8 a little.

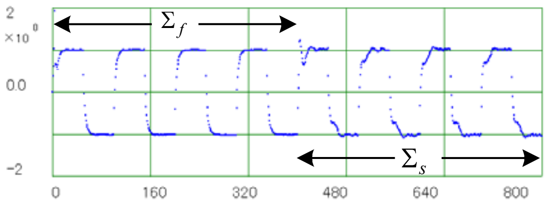


**Fig. 8** Controlled variable without tracking model

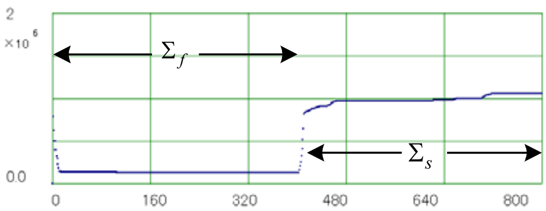


**Fig. 9** Weight $R$ without tracking model

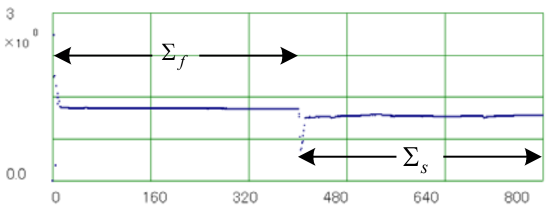


**Fig. 10** Stability index $\gamma$ without tracking model

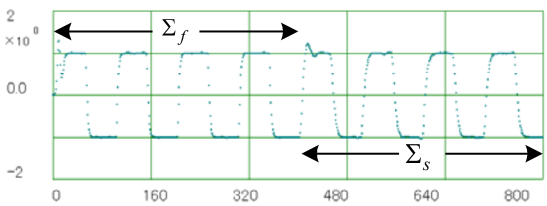


**Fig. 11** Controlled variable with tracking model

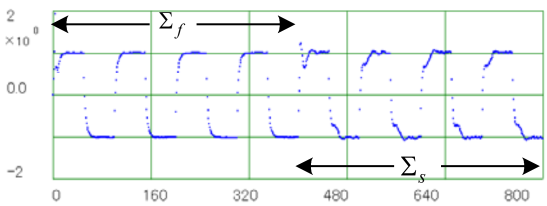


**Fig. 12** Actuating value with tracking model

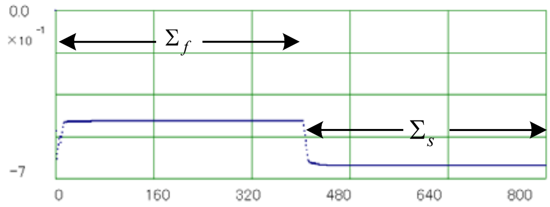


**Fig. 13** Tracking model pole $\alpha$ by proposed system

When the prefilter of (22) is specified and the tracking model of (17) is used, the transient response is finally improved as shown in Figs. 11 through 13. The controlled variable shows a good transient response in Fig. 11 and also the overshoot disappears after the completion of parameter identification, simultaneously an actuating value is suitably given without the disturbance as shown in Fig.12. Furthermore, the stable pole $\alpha$ is appropriately tuned as shown in Fig.13. The performance weight and the index are almost similar to Figs. 9 and 10 in this experiment.

## 6. Conclusions

This paper has proposed a useful strategy for achieving the adaptive control of DC motor speed system with great variation of load inertia. The difficulty of conventional adaptive pole placement control is how to place the stable pole in the *Diophantine* equation for designing the appropriate controller according to the load inertia of DC motor that widely changes. The proposed methods can place the stability index in the specified area and then, overcome the problem of unstable series compensator that appears in conventional adaptive control system. The evident solution for the control purpose is easily achieved by applying the proposed scheme. In conclusion the proposed method is sufficiently applicable to practical DC motor system.

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