A design of the DC motor control circuit

Abstract: The phase compensation method is applied in the control system design, the speed of DC motor is controlled by using the feedback of both armature current and tachometer voltage.

1. Modeling a DC motor

The symbols used in the modeling is defined as follows:

L : armature inductance $[H]$
R_a : armature resistance $[\Omega]$
i : armature current $[A]$
e : applied armature voltage $[V]$
K_e : back electromotive force constant $[V\sec/rad]$
K_t : torque constant $[Nm/A]$
J_m : rotor momentum of inertia $[kgm^2]$
w : rotor speed $[rad/\sec]$
J_ℓ : load momentum of inertia $[kgm^2]$
B_ℓ : viscous friction coefficient $[Nm/rad/\sec]$
t_f : disturbance torque $[Nm]$.

In this case the electrical equation is given as

$$L\frac{di}{dt}+R_a i=e-K_e w \tag{1.1}$$

and for the mechanical part of the system the torque equation is described as follows:

$$(J_m+J_\ell)\frac{dw}{dt}+B_\ell w=K_t i-t_f \quad . \tag{1.2}$$

It is often possible to neglect them on the design of a DC motor control system, because the effect of both the inductance L of Eq. (1.1) and the viscous torque $B_\ell w$ of Eq. (1.2) is very slight. The following block diagram for the design of a DC motor control system can be obtained.

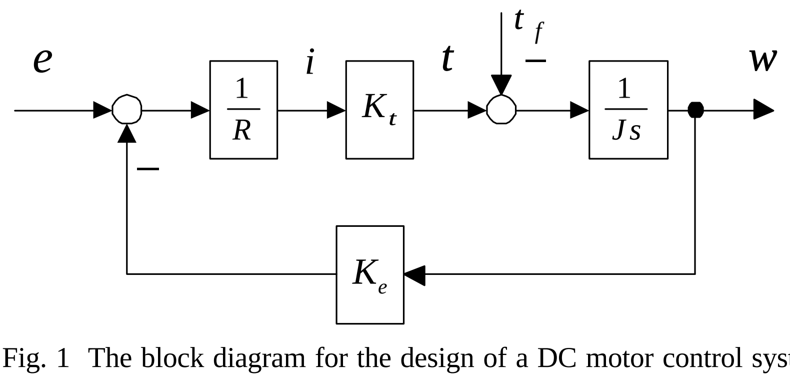

Fig. 1 The block diagram for the design of a DC motor control system.

The transfer function to the motor speed w from the applied armature voltage e in Fig. 1 is described as

$$\frac{w(s)}{e(s)}=\frac{K}{1+T_m s} \quad , \tag{1.3}$$

where the time constant T_m and the gain K are the following formulation:

$$T_m=\frac{(J_m+J_\ell)R_a}{K_t K_e} \quad , \quad K=\frac{1}{K_e} \quad . \tag{1.4}$$

2. Torque control by means of armature current feedback

In case of detection of the large current, a eddy current sensor is often used. It is detected by inserting the small resistance R_i in the armature circuit, because the motor current is not so large in this system. Here, let us use the symbol e_i as the current reference value. Then, G_i is the transfer function of the current amplifier. Moreover, when K_i is the feedback ratio of the motor current, the block diagram of the torque control system is shown by the following Fig. 2.

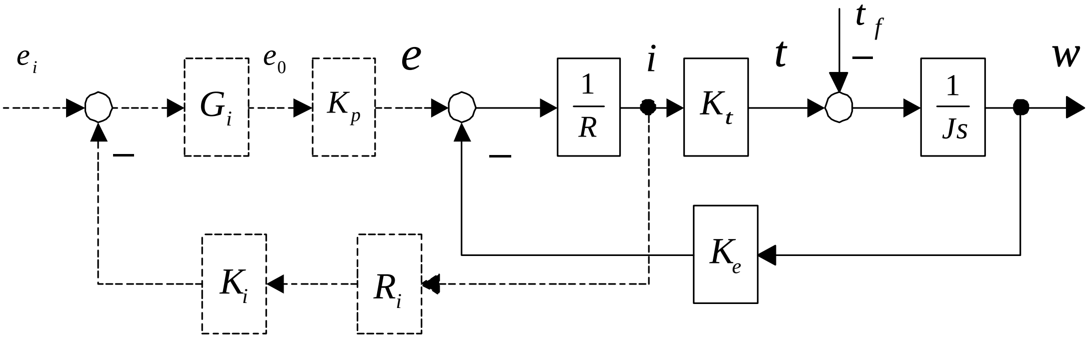

Fig. 2 The block diagram of the torque control system.

2.1 The construction of the current amplifier

The two typical current amplifiers shown by the following transfer function are considered:

A) Amplifier of the first order delay type:

$$G_i = \frac{K_{11}}{1+Ts} \tag{2.1}$$

B) Amplifier of the integral type:

$$G_i = \frac{K_{11}(1+Ts)}{s} \tag{2.2}$$

2.2 The case of using amplifier of the first order delay type

The transfer function to the motor speed w from the current reference value e_i in Fig. 2 is described as

$$\frac{w(s)}{e_i(s)} = \frac{K_{11}K_pK_t}{JRTs^2 + (JR + JK_{11}K_pK_iR_i + K_tK_eT)s + K_tK_e} \quad , \tag{2.3}$$

where J and R are given by the following relation:

$$J = J_m + J_l \quad , \quad R = R_a + R_i \quad . \tag{2.4}$$

In addition, the block diagram of the torque control system in the condition that the rotor was locked is shown by Fig. 3.

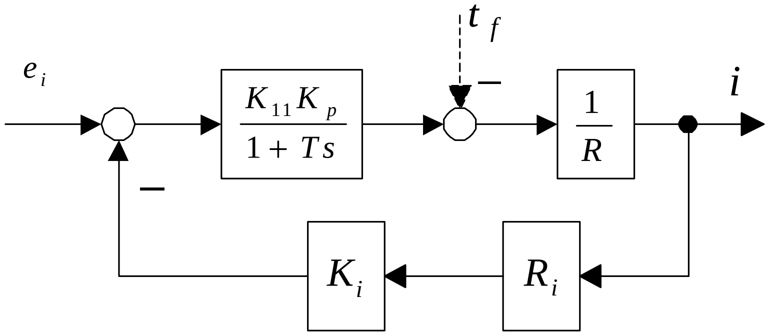

Fig. 3 The block diagram of the torque control system in the condition that the rotor was locked (in the case of using amplifier of the first order delay type).

The transfer function to the armature circuit current i from the current reference value e_i in Fig. 3 is described as

$$\frac{i(s)}{e_i(s)} = \frac{K_{11}K_p}{R + K_{11}K_pK_iR_i + RTs} \quad , \tag{2.5}$$

then the steady state armature current i is given as

$$i_s = \frac{K_{11}K_p}{R + K_{11}K_pK_iR_i} e_i \quad . \tag{2.6}$$

2.3 The case of using amplifier of integral type

The transfer function to the motor speed w from the current reference value e_i in Fig. 2 is described as

$$\begin{aligned}\frac{w(s)}{e_i(s)} &= \frac{K_{11}K_pK_t(1+Ts)}{s[(JR + JK_{11}K_pK_iR_iT)s + (JK_{11}K_pK_iR_i + K_tK_e)]} \\ &= \frac{K_{11}K_p(1+Ts)}{s} \cdot \frac{K'}{1+T_m' s} \quad ,\end{aligned} \tag{2.7}$$

where K' and T_m' are given as

$$K' = \frac{K_t}{JK_{11}K_pK_iR_i + K_tK_e} \quad , \quad T_m' = \frac{J(R + K_{11}K_pK_iR_iT)}{JK_{11}K_pK_iR_i + K_tK_e} \quad . \tag{2.8}$$

At this time, if the designer selects the relation $T = T_m'$, then the motor speed has the characteristics of an integral element to the reference value. In addition, the block diagram of the torque control system in the condition that the rotor was locked is shown by Fig. 4.

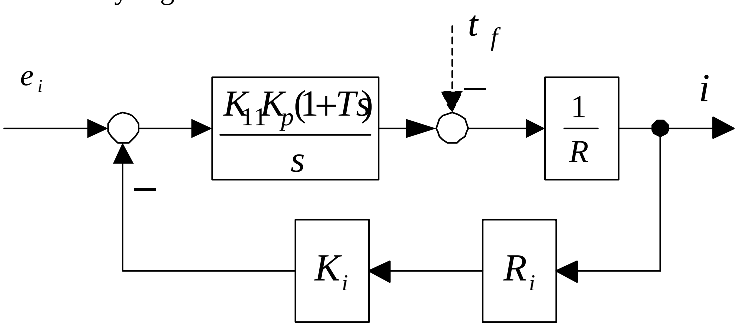

Fig. 4 The block diagram of the torque control system in the condition that the rotor was locked

The transfer function to the armature circuit current i from the current reference value e_i in Fig. 4 is described as

$$\frac{i(s)}{e_i(s)} = \frac{K_{11}K_p(1+Ts)}{K_{11}K_pK_iR_i + (K_{11}K_pK_iR_iT + R)s} \quad , \tag{2.9}$$

then the steady state armature current i is given as

$$i_s = \frac{1}{K_iR_i} e_i \quad . \tag{2.10}$$

3. Speed control by means of tacho-generator voltage feedback

In case of detection of the motor speed, a tacho-generator is very often used. On the other hand, the optical encoder is generally used when the position control system is designed. Here, let us use the tacho-generator as the speed sensor. Then, G_v is the transfer function of the speed amplifier. Moreover e_w and S_v are the speed reference value and the tacho-generator constants, respectively. The block diagram of the speed control system that the tacho-generator voltage is feedbacked at the outer loop of the current feedback loop is shown by the following Fig. 5.

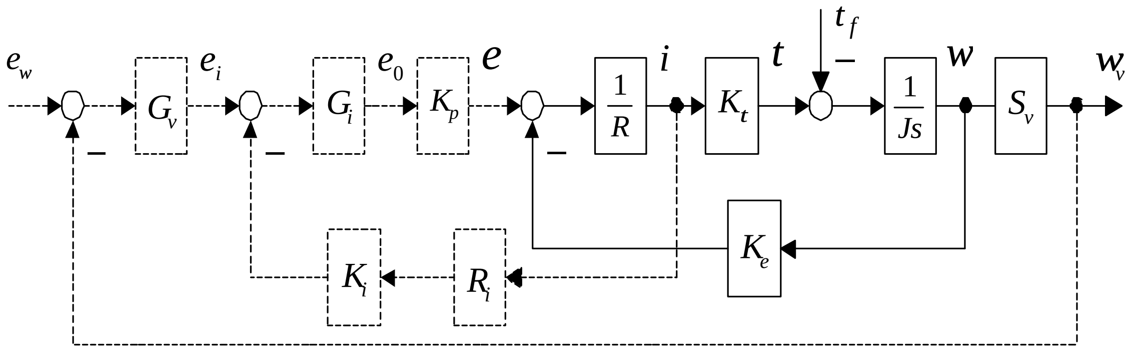

Fig. 5 The block diagram of the speed control system.

The transfer function of a speed amplifier is designed respectively as the different form by means of either the first order delay type or the integral type as the current amplifier. In every case, the lag compensator is designed in order to increase the gain margin in the low frequency range

4. Outline of motor controller

4.1 Major constants of DC motor

The constants used in the design of a motor controller are shown in the following table 1.

4.2 Power amplifier of motor driver

The power amplifier circuit of the motor driver that is constructed by cascading of two power operational amplifiers is shown by Fig. 6.

Table 1 The design constant (DC motor TYPE SS40E2-E SAWAMURA DENKI CO.LTD)

Torque constant	$K_t = 7.154 \times 10^{-2} [Nm/A]$
Back electromotive force constant	$K_e = 7.162 \times 10^{-2} [V\sec/rad]$
Armature resistance	$R_a = 4.3[\Omega]$
Rotor momentum of inertia	$J_m = 0.4 \times 10^{-4} [kgm^2]$
Conversion constant w to w_v	$S_v = 3.183 \times 10^{-2} [V/rad/\sec]$
Power amplifier gain	$K_p = 2$
Resistance for current detection	$R_i = 0.2[\Omega]$

----- Conversion example of physical unit -----

The case of torque constant given by motor specification $K_t = 0.73 [kgf \cdot cm/A]$:

$$K_t = 0.73 \times 9.8 \times 10^{-2} = 7.154 \times 10^{-2} [Nm/A]$$

The case of back electromotive force constant given by motor specification $K_e = 7.5 [V/Krpm]$:

$$K_e = 7.5 \times \frac{60}{2p \times 1000} \quad 7.162 \times 10^{-2} [V\sec/rad]$$

The case of conversion constant w to w_v given by F/V converter specification $10[Volt]per100[kHz]$:

$$S_v = \frac{Pf}{2p} \cong 3.183 \times 10^{-2} [V/rad/\sec], \ P = 2000[pulse/rev], \ f = 1 \times 10^{-4}$$

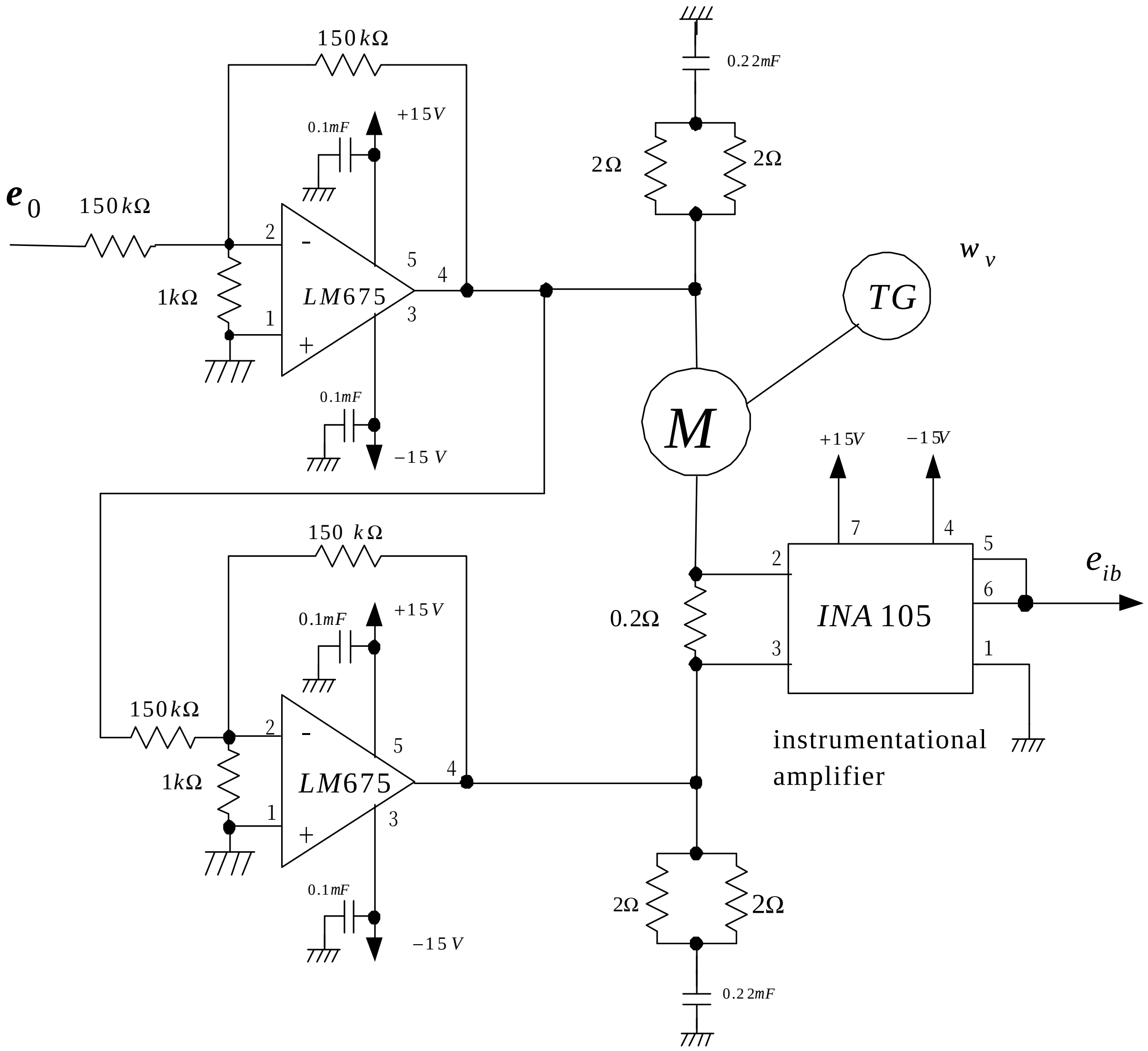

Fig. 6 Power amplifier circuit by using two power operational amplifier LM675

5. Design procedure of controller

5.1 **Designing the current amplifier of the first order delay type**

The output voltage of the instrumentation amplifier(IN105KP) in the power amplifier circuit (see Fig.6) is used as the current feedback signal in torque control. The design specification of this type is as follows:

1. Steady current i_s to the current reference value $e_i = 0.1$[volt] is about 0.20 to 0.25[A].
2. Gain K_{11} of current amplifier is specified by the range of 20 to 30 on the case of the one chip mount.
3. Time constant T is selected as about 0.001 to 0.01 in order to move up the rising time of armature current i.

Here, the steady current i_s to the current reference value $e_i = 0.1$[volt] is chosen as 0.20[A] in consideration of the design specification 1. The design example of using both $K_{11} = 30$ and $T = 0.001$ is described below. When these constants are substituted to Eq. (2.6), the feedback ratio K_i is obtained as $K_i = 2.125$ by solving the following relation:

$$0.20 = \frac{30 \times 2}{4.5 + 30 \times 2 \times K_i \times 0.2} \times 0.1 \quad . \tag{5.1}$$

Therefore, the current amplifier of the first order delay type is given by the following transfer function:

$$G_i = \frac{30}{1 + 0.001s} \quad . \tag{5.2}$$

The circuit examples implemented by means of the operational amplifier are shown by Figs 7 and 8.

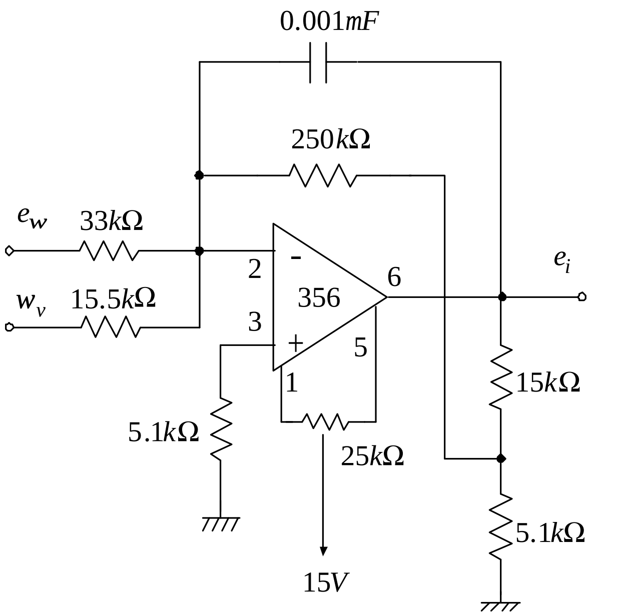

Fig. 7 The current amplifier of first order delay type.

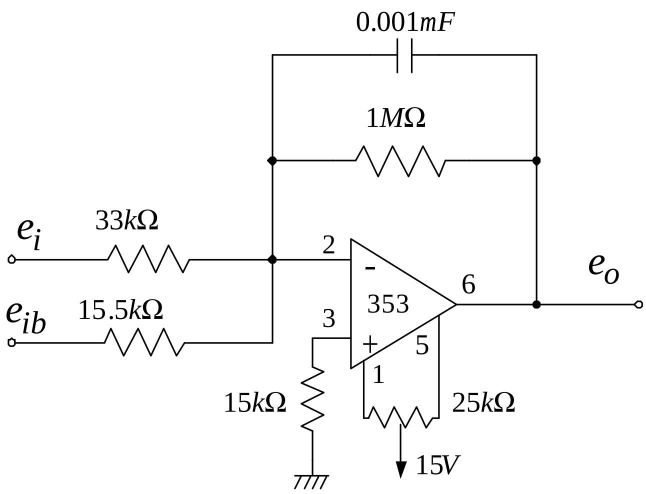

Fig. 8 The current amplifier of first order delay type.

At this time, the second order term concerning Laplace operator s is negligible, because this term is sufficiently small than the first order term. Therefore, the transfer function to the motor speed w_v from the current reference value e_i can be simplified as

$$G_o = \frac{w_v\ (s)}{e_i(s)} = \frac{K_{11} K_p K_t S_v}{JRTs^2 + (JR + JK_{11} K_p K_i R_i + K_t K_e T)s + K_t K_e}$$

$$\cong \frac{K}{1+T_r s} \quad , \tag{5.3}$$

where K_o and T_r are given as

$$K_o = \frac{K_{11} K_p S_v}{K_e} \quad , \quad T_r = \frac{JR + JK_{11} K_p K_i R_i + K_t K_e T}{K_t K_e} \quad . \tag{5.4}$$

Furthermore, the transfer function model for the design is derived under the optimal load $J = 1.5 J_m$, because there are few cases a motor is driven with a no-load. When each constant defined before is substituted to Eq. (5.4), the transfer function is derived by the following equation:

$$K_o = \frac{30 \times 2 \times 3.183 \times 10^{-2}}{7.162 \times 10^{-2}} \cong 26.7 \quad , \tag{5.5}$$

$$T_r = \frac{0.60 \times 10^{-4} (4.5 + 30 \times 2 \times 2.125 \times 0.2) + 7.154 \times 10^{-2} \times 7.162 \times 10^{-2} \times 10^{-3}}{7.154 \times 10^{-2} \times 7.162 \times 10^{-2}} \cong 0.352 \quad . \tag{5.6}$$

5.2 Design of the speed amplifier

The output voltage of the tacho-generator in the power amplifier circuit (see Fig.6) is used as the feedback signal in the speed control. The design specification of this speed amplifier is as follows:

1. The speed amplifier involved the integral characteristics is chosen in order to reject a stationary disturbance and it is given by the following transfer function :

$$G_v = \frac{K_2 (1 + T_r s)}{s} \cdot \frac{1 + T_2 s}{1 + T_1 s} \quad . \tag{5.7}$$

2. The gain-crossover angular frequency w_m of the loop transfer function in the speed control system is specified by the range 100 to 150.
3. The phase margin q_m of a speed control system is selected as about 60 to 65 degree.

At this time, the total gain $K_0 K_2$ is chosen as 240 in order to meet the gain-crossover angular frequency w_m. Due to $K_0 = 26.7$ in Eq. (5.5), the gain K_2 of the speed amplifier yields 8.99. When the compensator to fill the phase margin q_m of 60 degree is derived by using the phase compensation method, the gain-crossover angular frequency w_m yields 138 [rad/sec] and the speed amplifier G_v is given by the following transfer function:

$$G_v = \frac{8.99(1 + 0.352 s)}{s} \cdot \frac{1 + s/240}{1 + s/80} \quad . \tag{5.8}$$

5.3 Designing the current amplifier of the integral type

The design specification of this type is as follows:

1. Steady current i_s to the current reference value $e_i = 0.1$ [volt] is about 0.5[A].
2. Gain K_{11} of current amplifier is specified by the range of 20 to 30 on the case of the one chip mount.
3. Time constant T is selected as $T \cong T'_m$ so that the transfer function of e_i to w possesses the approximate integral characteristics.

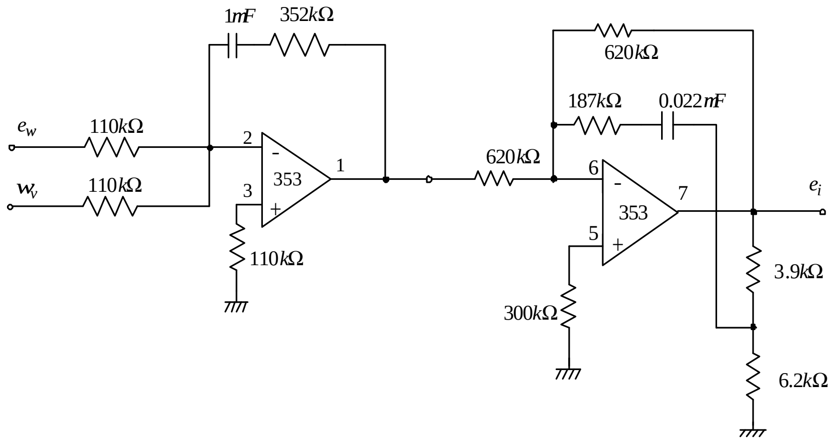

Fig. 9 The speed amplifier (in the case of the current amplifier of first order delay type).

5.3 **Designing the current amplifier of the integral type**

The design specification of this type is as follows:

1. Steady current i_s to the current reference value $e_i = 0.1$[volt] is about 0.5[A]. 2. Gain K_{11} of current amplifier is specified by the range of 20 to 30 on the case of the one chip mount 3. Time constant T is selected as $T \cong T'_m$ so that the transfer function of e_i to w possesses the approximate integral characteristics.

Here, the steady current i_s to the current reference value $e_i = 0.1$[volt] is chosen as 0.5[A] in consideration of the design specification 1. The design example of using $K_{11} = 30$ is described below. When these constants are substituted to Eq. (2.10), the feedback ratio K_i is obtained as $K_i = 1.0$ by solving the following relation:

$$0.5 = \frac{1}{K_i \times 0.2} \times 0.1 \quad . \tag{5.9}$$

At this time, the transfer function G_o to the motor speed w_v from the current reference value e_i can be given by as follows:

$$G_o = \frac{w_v\ (s)}{e_i(s)} = \frac{K_{11}K_pK_t(1+Ts)S_v}{s[(JR + JK_{11}K_pK_iR_iT)s + (JK_{11}K_pK_iR_i + K_tK_e)]} = \frac{K_0(1+Ts)}{s} \cdot \frac{1}{1+T'_m s}, \tag{5.10}$$

where K_0 and T'_m are given as

$$K_0 = \frac{K_{11}K_pK_tS_v}{JK_{11}K_pK_iR_i + K_tK_e} \quad , \quad T'_m = \frac{J(R + K_{11}K_pK_iR_iT)}{JK_{11}K_pK_iR_i + K_tK_e} \quad . \tag{5.11}$$

Furthermore, the transfer function model for the design is derived under the valid load $J = 1.5J_m$ similarly to the design parameter of section 5.1. When each constant defined before is substituted to K_0 of Eq. (5.11), K_0 is obtained by the following equation:

$$K_o = \frac{30 \times 2 \times 7.154 \times 10^{-2} \times 3.183 \times 10^{-2}}{0.60 \times 10^{-4} \times 30 \times 2 \times 1 \times 0.2 + 7.154 \times 10^{-2} \times 7.162 \times 10^{-2}} \cong 23.38 \quad , \tag{5.12}$$

In addition, T'_m of Eq. (5.11) is derived as $T'_m \cong 0.0527$ by solving the following relation:

$$T'_m = \frac{0.6 \times 10^{-4}(4.5 + 30 \times 2 \times 1 \times 0.2 \times T'_m)}{0.6 \times 10^{-4} \times 30 \times 2 \times 1 \times 0.2 + 7.154 \times 10^{-2} \times 7.162 \times 10^{-2}} \quad . \tag{5.13}$$

Therefore, when T is decided as 0.0527 by the design specification 3, the current amplifier G_i of the integral type is given by the following transfer function:

$$G_i = \frac{30(1+0.0527s)}{s} \tag{5.14}$$

The circuit examples implemented by means of the operational amplifier are shown by Fig. 10.

At this time, the transfer function to G_o the motor speed w_v from the current reference value e_i can be simplified as the following design model:

$$G_o = \frac{w_v(s)}{e_i(s)} = \frac{23.38}{s} \tag{5.15}$$

5.4 **Design of the speed amplifier**

The design specification of this speed amplifier is as follows:

1. A speed amplifier is given by the following characteristic in order to apply the phase compensation method :
$$G_v = \frac{K_2(1+T_2 s)}{1+T_1 s} \quad . \tag{5.16}$$
2. The gain-crossover angular frequency w_m of the loop transfer function in the speed control system is specified by the range 100 to 150.
3. The phase margin q_m of a speed control system is selected as about 60 to 65 degree.

At this time, the total gain K_0K_2 is chosen as 200 in order to meet the gain-crossover angular frequency w_m. Due to $K_0 = 23.38$ in Eq. (5.5), the gain K_2 of the speed amplifier yields 8.55. When the compensator to fill the phase margin q_m of 60 degree is derived by using the phase compensation method, the gain-crossover angular frequency w_m yields 110 [rad/sec] and the speed amplifier G_v is given by the following transfer function:

$$G_v = \frac{8.55(1+s/190)}{1+s/63.5} \quad . \tag{5.8}$$

The circuit examples concerning the speed amplifier implemented by means of OP.amplifier are shown by Fig. 11.

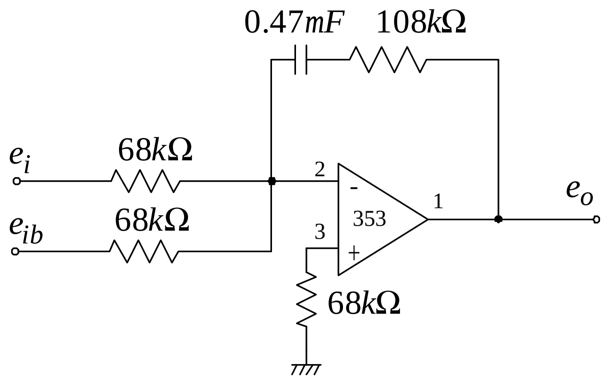

Fig. 10 The current amplifier of integral type.

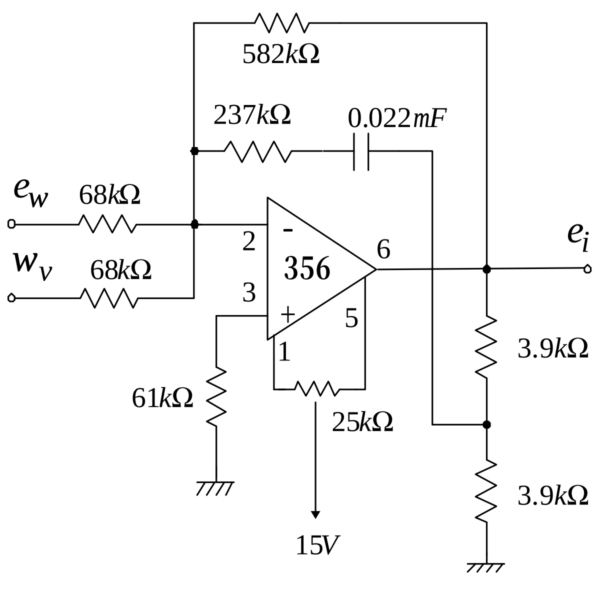

Fig. 11 The speed amplifier (in the case of the current amplifier of integral type).