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# Auto-Tuning for Strongly Stable Adaptive Pole Placement Control System

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### Abstract

The auto-tuning method for a strongly stable adaptive control system design is proposed. The proposed methods can place the stability index in the specified area and then, overcome the problem of unstable series compensator that appears in conventional adaptive pole placement control system. The appropriate adaptive control system can be derived by adjusting automatically the weight of a performance criterion in optimal control by means of the fuzzy inference on the basis of the stability index. Furthermore, the transient characteristic is improved by tuning the tracking model according to certain relation between the performance weight in an optimal control design, the settling time and the tracking model pole. In addition, the numerical simulations are used to prove that the proposed methods provide satisfied performance.

## 1 Introduction

The dynamic characteristics in many industrial controlled systems often change in a wide range during actuation. For example, the mechanical parameter on the motor control system changes over a wide range. It is well known that the adaptive control [1], [2], is very effective method for such systems. By the way, when the system parameter changes widely, a controller having unstable poles frequently appears in an adaptive pole placement control system, even if the closed-loop characteristic has been designed as a stable system. However, the appearance of unstable compensator is not desirable with respect to both stability and reliability.

The unstable controller is used seldom, especially if the plant itself is a stable system. Therefore, the appropriate selection of a closed-loop pole is required in order to obtain a stable controller over a full range of the parameter change. However, selecting the closed-loop pole that ensures the stability of a compensator under the condition of the parameter change over a wide range is very difficult in the pole placement control. Therefore, if the designer is able to adjust the stable poles of a closed-loop system recursively according to the change of the plant parameter, the construction of a strongly stable system [3] is easily realized. Such research is an indispensable significant theme to the development of an intelligent auto-tuning technology. However, only few reports are published that discussed the tuning method of both controller and closed-loop pole according to the parameter change.

Recently, authors announced some effective design methods [4], [5], that construct a strongly stable adaptive pole placement system when the plant parameter greatly changes. In this paper, the desired performance of both transient response and manipulated variable can be achieved by tuning the tracking model in consideration of the relation between the performance weight in optimal control design and the settling time. The main characteristic of the proposed design method for strongly stable adaptive control system is to evaluate the control system by introducing a stability index, the relative stability of both series compensator and closed-loop system. A stability index is the evaluation that was introduced in the coefficient diagram method [6], [7], it is also known as a useful index in the case that a robust controller of low dimension is derived in the control design.

The procedure of the proposed method is summarized as follows. An optimal servo system [8] is recursively designed to an estimated plant model. The pole placement controller is automatically constructed by solving *Bezout* identity on the basis of the characteristic polynomial of the derived optimal servo. After the stability index of a series compensator is examined, the weight in an optimal design is appropriately updated by means of the fuzzy inference.

In other words, the proposed method automatically adjusts the weight of an optimal servo system so that the adaptive system can place the stability index of the series compensator into the specified region. Consequently, this method not only ensures the stability of both the closed-loop system and the controller but also can achieve a fine control performance.

## 2 Adaptive Pole Placement Control System

The proposed adaptive control system shown in Figure 1 does not introduce the pole-zeros cancellation because it is really difficult to avoid the unstable zeros in the discrete modeling of the practical plant.

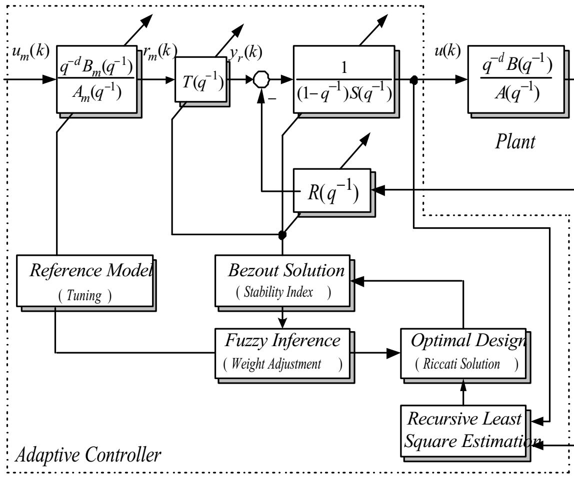


**Figure 1:** Block diagram of an adaptive control system.

### 2.1 Pole placement control system

The adaptive control system of this research is constructed by means of the pole placement design with an integrator in order to reject the stationary disturbance. The plant is described by the following ARX model:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + w(k) \ , \tag{1}$$

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n} \ , \tag{2}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_mq^{-m} \ , \tag{3}$$

where $w(k)$ is the white noise having zero mean value. The reference signal is given by the following tracking model:

$$A_m(q^{-1})r_m(k) = q^{-d}B_m(q^{-1})u_m(k) \tag{4}$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + \cdots + a_{ml}q^{-l} \ , \tag{5}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \cdots + b_{ml}q^{-l} \ . \tag{6}$$

The regulation performance of a stable closed-loop system is specified by the following polynomial:

$$D_o(q^{-1}) = 1 + d_1q^{-1} + \cdots + d_{nd}q^{-nd} \ , \ nd \le n + m + d \ . \tag{7}$$

In addition, the tracking performance of a control system is generally achieved by using prefilter $T(q^{-1})$ that is either the constant gain such as

$$T(q^{-1}) = D_o(1)/B(1) \tag{8}$$

or the polynomial such as

$$T(q^{-1}) = D_o(q^{-1})/B(1) \ . \tag{9}$$

Here, let us consider the difference described by

$$e(k+d) = D_o(q^{-1})y(k+d) - T(q^{-1})B(q^{-1})r_m(k+d) \ . \tag{10}$$

The variance of (10) represents the performance criterion:

$$J = E[e^2(k+d)] \ . \tag{11}$$

Then, the optimal input signal $u(k)$ that minimizes $J$ is constructed by means of the following value:

$$u(k) = \frac{T(q^{-1})r_m(k+d) - R(q^{-1})y(k)}{(1-q^{-1})S(q^{-1})} \ , \tag{12}$$

where $S(q^{-1})$ and $R(q^{-1})$ are given by the polynomials:

$$S(q^{-1}) = 1 + s_1q^{-1} + \cdots + s_{ns}q^{-ns} \ , \ ns = m + d - 1 \ , \tag{13}$$

$$R(q^{-1}) = r_0 + r_1q^{-1} + \cdots + r_{nr}q^{-nr} \ , \ nr = n \ . \tag{14}$$

Furthermore, $S(q^{-1})$ and $R(q^{-1})$ can be derived by solving *Bezout* identity (or *Diophantine* equation):

$$D_o(q^{-1}) = (1-q^{-1})A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) \ . \tag{15}$$

### 2.2 Recursive identification system

When the parameter of ARX model (1) is unknown or greatly varied, the estimated value of a plant parameter is substituted in the control system design. If the parameter vector $\theta$ and the regression vector $\varphi(k)$ constructed by measurement data are defined as

$$\theta^T = \{a_1, a_2, \cdots, a_n, b_0, b_1, \cdots, b_m\} \tag{16}$$

and

$$\varphi^T(k) = \{-y(k-1), -y(k-2), \cdots, -y(k-n), \\ u(k-d), u(k-1-d), \cdots, u(k-m-d)\} \ , \tag{17}$$

respectively, then, the plant output is expressed by

$$y(k) = \varphi^T(k)\theta + v(k) \ . \tag{18}$$

Here, let us use the notation $\hat{y}(k\,|\,\theta)$ as the one-step-ahead prediction value, then it is given by the linear formula with respect to a parameter vector $\theta$ as follows:

$$\hat{y}(k\,|\,\theta) = [1 - A(q^{-1})]y(k) + B(q^{-1})u(k) = \varphi^T(k)\theta \ . \tag{19}$$

The estimate model output is constructed according to

$$y_m(k) = \varphi^T(k)\hat{\theta}(k) \ , \tag{20}$$

where the estimate vector of the parameter is defined by

$$\hat{\theta}^T(k) = \{a_1(k), a_2(k), \cdots, a_n(k), \\ b_0(k), b_1(k), \cdots, b_m(k)\} . \tag{21}$$

If the condition of signal to noise (S/N) ratio of the plant described by ARX model is good, the least-squares estimation is reliable and has few bias with respect to the estimated value. Consequently, the recursive identification based on least-squares method is described by means of the following formulation:

**parameter adjusting:**

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{1 + \varphi^T(k)P(k-1)\varphi(k)}\varepsilon(k) \ , \tag{22}$$

**adaptive gain:**

$$P(k) = \frac{1}{\lambda_1(k)}\left\{P(k-1) - \frac{\lambda_2(k)P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda_1(k) + \lambda_2(k)\varphi^T(k)P(k-1)\varphi(k)}\right\}, \tag{23}$$

**apriori error:**

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \ , \tag{24}$$

where weighting sequences $\lambda_1(k)$ and $\lambda_2(k)$ in (23) are $0 < \lambda_1(k) \le 1$ and $0 \le \lambda_2(k) < 2$, respectively. The designer can obtain other adaptive gain that has the typical characteristic by selecting the appropriate values for $\lambda_1(k)$ and $\lambda_2(k)$.

### 2.3 Characteristic polynomial based on optimal servo

The procedure of a tuning algorithm was simplified by applying the binomial coefficient polynomial in [4], as the regulation specification. Type-1 optimal servo base on the

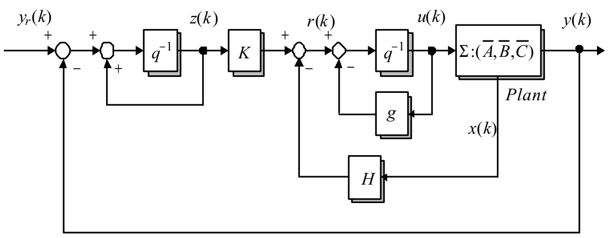


**Figure 2:** Type-1 optimal servo having one sample delay.

state-space approach is adaptively updated. The state-space description, for example, a controllable canonical form with respect to ARX model (1) is given by $\Sigma:(\overline{A},\overline{B},\overline{C})$ ignoring the disturbance $w(k)$. Type-1 servo system with computation time delay to the plant $\Sigma:(\overline{A},\overline{B},\overline{C})$ as shown in Figure 2 is constructed.

Here, a quadratic type performance criterion is defined as

$$J=\sum_{k=0}^{\infty}\{\tilde{x}^T(k)Q\tilde{x}(k)+R\tilde{v}^2(k)\}\ ,\quad R>0\ , \tag{25}$$

where $\tilde{x}(k)$ and $\tilde{v}(k)$ represent the states and actuating value for the extended deviation system, respectively, and Q is a semi-positive definite matrix. Type-1 optimal servo having one sample controller delay that minimizes a performance index of (25) is given as follows:

***Riccati* equation:**

$$P=Q+\overline{A}^T P\overline{A}-\overline{A}^T P\overline{B}(R+\overline{B}^T P\overline{B})^{-1}\overline{B}^T P\overline{A},\ P>0\ , \tag{26}$$

**Optimal feed-back gain:**

$$F=(R+\overline{B}^T P\overline{B})^{-1}\overline{B}^T P\overline{A}\ , \tag{27}$$

**Controller parameters:**

$$g=F\overline{B}+1\ , \tag{28}$$

$$[H,K]=[F\overline{A}^2, F\overline{A}\overline{B}+F\overline{B}+1]E^{-1}\ , \tag{29}$$

$$E=\begin{bmatrix}\overline{A}-I & \overline{B}\\ \overline{C} & 0\end{bmatrix}, \tag{30}$$

Furthermore, the state-space description of this optimal servo system is expressed by

$$\begin{bmatrix}x(k+1)\\u(k+1)\\z(k+1)\end{bmatrix}=\begin{bmatrix}\overline{A} & \overline{B} & 0\\ -H & -g & K\\ -\overline{C} & 0 & 1\end{bmatrix}\begin{bmatrix}x(k)\\u(k)\\z(k)\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}r(k)\ . \tag{31}$$

Therefore, characteristic polynomial $D_0(q^{-1})$ of closed-loop system is, for example, recursively calculated by *Faddeev*'s algorithm from the system matrix of (31).

### 2.4 Auto-Tuning based on the stability index

The tuning method for the strongly stable pole placement based on the stability index is discussed. The stability index is introduced in order to evaluate the relative stability of the control system. In the following $n^{th}$-order characteristic polynomial of the continuous-time transfer function:

$$p(s)=f_n s^n+\cdots+f_1 s+f_0\ ,\quad n\ge 2\ , \tag{32}$$

the stability indices $\gamma_i$ are generally defined as follows:

$$\gamma_i=f_i^2/f_{i+1}\cdot f_{i-1}\ ,\quad (i=1,\cdots,n-1)\ . \tag{33}$$

A series compensator $[S(q^{-1})]^{-1}$ is derived by solving (15), according to the characteristic polynomial $D_o(q^{-1})$ that is obtained with an optimal design of the previous section.

Next, in order to apply the stability indices $\gamma_i$ to the discrete-time series compensator $[S(q^{-1})]^{-1}$, it is transformed into the continuous-time transfer function $S_c(s)$ by introducing the inverse bilinear transformation

$$q=(2+Ts)/(2-Ts)\ , \tag{34}$$

where $T$ is the sampling period.

After this operation, the stability indices $\gamma_i$ in terms of the denominator of continuous-time compensator $S_c(s)$ are calculated. Furthermore, the simplest index $\gamma$ is selected by means of the algebraic product (35) of stability indices $\gamma_i$ in order to evaluate the relation between the stability of controller and the performance of closed-loop system.

$$\gamma=\prod_{i=1}^{n-1}\gamma_i\ . \tag{35}$$

This new index $\gamma$ can be related to the performance weight $R$ in the optimal servo. Furthermore, the index $\gamma$ is related to each characteristic, whether open-loop, such as gain-phase margin, or close-loop, such as settling time. Appropriate auto-tuning of weight $R$ based on the resultant stability index $\gamma$ is achieved, by considering the relations as mentioned above. Namely, the fuzzy inference is introduced to adjust the performance weight $R$ and this stability index $\gamma$ is effectively used as the scaling factor.

In addition, in order to complete the defuzzification of the inference result, the consequent of fuzzy inference is executed using the gravity method of *Mamdani* as follows:

$$u_g(k-1)=\{\sum x_j\cdot\mu(x_j)\}/\sum\mu(x_j)\ , \tag{36}$$

where $x_j$ is the nonfuzzy value, $\mu(x_j)$ is the value of the membership function, and $u_g(k-1)$ is the inference result. In order to place the stability index $\gamma$ of the series compensator $S_c(s)$ into the specified region, the weight $R$ of a performance criterion in the optimal servo system is tuned by means of the following recursive formula:

$$R(k)=R(k-1)\cdot 10^{\beta(k-1)}\ , \tag{37}$$

$$\beta(k-1)=c\cdot u_g(k-1)\ ,\quad c{:}\,any\,constant\ . \tag{38}$$

## 3 Design Example

A $3^{rd}$-order continuous-time plant is chosen as the controlled process according to the following:

$$G(s)=\frac{K\omega_n^2}{(s+d)(s^2+2\varsigma\omega_n s+\omega_n^2)}\ , \tag{39}$$

where $K$ and $d$ are assigned to a constant value of 50 in order to simplify the calculation similarly to [5]. The design is performed according to the procedure described below. The controllers for both of the continuous-time plant pairs, $\Sigma_f(\varsigma=0.1,\omega_n=60)$ and $\Sigma_s(\varsigma=0.4,\omega_n=20)$, are designed. The plant $\Sigma_f$ is a typical plant that has a fast response, and also the plant $\Sigma_s$ is a typical plant, yet has a slower response than the plant $\Sigma_f$. First the plant $\Sigma_s$ described above is transformed with a sampling time $T=0.02$[sec] into the following discrete-time system (40), because the digital controller is designed using the discrete-time model.

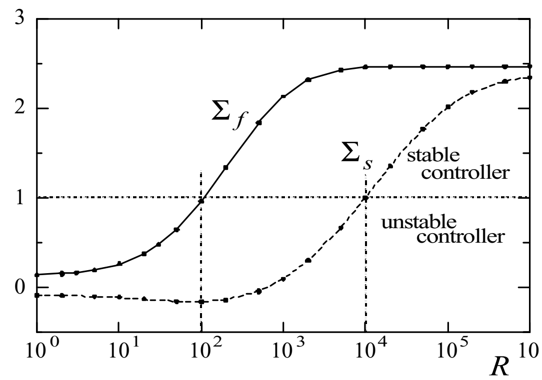


**Figure 3:** Each index $\gamma$ of plants $\Sigma_f$ and $\Sigma_s$ to weight $R$.

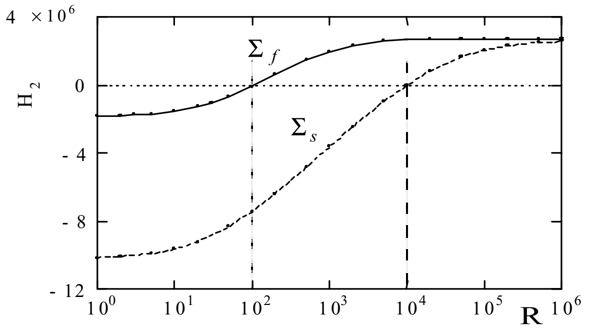


**Figure 4:** Each *Hurwitz* determinant $H_2$ to the weight $R$.

$$G(q^{-1})=\frac{q^{-1}(0.019318+0.056083\,q^{-1}+1.0006\times10^{-2}q^{-2})}{1-1.9589q^{-1}+1.3115q^{-2}-0.26714q^{-3}} \quad . \quad (40)$$

Due to the computation time delay, $d$ should be increased by $1$, yielding $d=2$. Type-1 optimal servo controller having one sample controller delay is calculated under the condition that the weights of performance criterion of (25) are $Q=diag(100,100,100)$ and $R=5\times10^4$.

When *Riccati* equation of (26) is first solved, a positive definite solution is derived as follows:

$$P=\begin{bmatrix}5.4411\times10^2 & -1.7943\times10^3 & 1.6504\times10^3\\ -1.7943\times10^3 & 7.5100\times10^3 & -6.8676\times10^3\\ 1.6504\times10^3 & -6.8676\times10^3 & 7.1081\times10^3\end{bmatrix}. \quad (41)$$

Therefore, the optimal feed-back gain of (27) is given as

$$F=[3.3250\times10^{-2}\ \ -1.3433\times10^{-1}\ \ 1.2356\times10^{-1}]\ , \quad (42)$$

the controller parameters are obtained, respectively, as

$$g=1.1236, \quad K=1.2632,$$
$$H=[0.31278\ \ -1.1229\ \ 1.2313]. \quad (43)$$

At the same time, by applying to (31) for *Faddeev*'s algorithm, the characteristic polynomial $D_o(q^{-1})$ of an optimal servo system is calculated as follows:

$$D_0(q^{-1})=1-1.8354q^{-1}+1.1771q^{-2}-0.23389q^{-3}$$
$$-6.5198\times10^{-16}q^{-4}+4.5552\times10^{-17}q^{-5}, \quad (44)$$

The order $nd$ of characteristic polynomial is equal to 5 theoretically. However, it is actually possible to process the order of $D_o(q^{-1})$ as $nd=3$, because both coefficients of the fourth and fifth order term in (44) are very small. If the designer requests more closed-loop polynomial than third order, for example, the design should be considered by using the plant model with the virtual pole-zero pair. When (15) is solved after the characteristic polynomial $D_o(q^{-1})$ is substituted to it, the pole placement controllers $S(q^{-1})$ and $R(q^{-1})$ are derived, respectively, as

$$S(q^{-1})=1+1.1236q^{-1}+0.93477q^{-2}+0.14351q^{-3}, \quad (45)$$
$$R(q^{-1})=15.349-29.413q^{-1}+19.159q^{-2}-3.8313q^{-3}. \quad (46)$$

Furthermore, the stability index $\gamma_i (i=1,2)$ and the scaling factor $\gamma$ are obtained, respectively, as

$$\gamma_1=1.7316, \quad \gamma_2=1.0224 \quad , \gamma=1.7704 \ . \quad (47)$$

*Hurwitz* determinant $H_2$ is calculated as $H_2=1.647\times10^6$ in order to confirm the absolute stability of $S_c(s)$.

In addition, the gain and phase margins from the loop transfer function are calculated and also the settling time of only closed-loop system can be obtained. Similarly, when the calculation is repeated changing $R$ continuously, using fixed Q in terms of both weights, Q and $R$, in the performance criterion, each significant characteristic is obtained. First, the index $\gamma$ of series compensator for the weight $R$ of a performance criterion of both plants $\Sigma_f$ and $\Sigma_s$ is shown in Figure 3. Furthermore, *Hurwitz* determinant $H_2$ and settling time $ST$ to performance weight $R$ are shown, respectively, in Figures 4 and 5. The diagrams in terms of the characteristic of the gain-phase margin are omitted in this paper.

Next, the relation of both performance weight $R$ and stability index $\gamma$ having good performance for both plants $\Sigma_f$ and $\Sigma_s$ is examined, respectively, in these Figures 3 and 4. Each range of the performance weights that meet stability condition $H_2>0$ is estimated in Figure 4 as

$$10^2\le R_f \quad , \quad 10^4\le R_s \ , \quad (48)$$

where $R_f$ and $R_s$ represent the range of performance weight regarding the plants $\Sigma_f$ and $\Sigma_s$, respectively. Therefore, the region of the stability index having a stable compensator is given as $1<\gamma$ in consideration of (48) with respect to Figure 3. Furthermore, the performance weight having the appropriate settling time for both of plants, $\Sigma_f$ and $\Sigma_s$ is estimated in Figure 5, respectively, as

$$R_f\cong5\times10^2 \quad , \quad R_s\cong5\times10^4 \ . \quad (49)$$

At this time, the designer is able to choose the region flexibly as a suitable area of both stability and settling time by considering Figure 3. However, the selection of a wide range is not appropriate for the purpose of auto-tuning. For example, each region of the stability index corresponding to $R_f$ and $R_s$ is appropriately selected for auto-tuning according to the following:

$$\left.\begin{array}{lll} l_f\le\gamma_f\le u_f \ , & l_f=1.5 \ , & u_f=2.0\\ l_s\le\gamma_s\le u_s \ , & l_s=1.7 \ , & u_s=1.8\end{array}\right\} . \quad (50)$$

Thus, many degrees of freedom are given to the designer on the occasion of a decision of $l_f, u_f$ and $l_s, u_s$ in (50). Fuzzy inference is used to ensure the stability index $\gamma$ of the series compensator within the specified region.

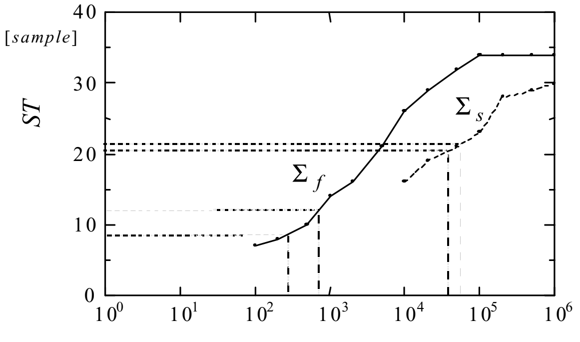


**Figure 5:** Each settling time $ST$ to the weight $R$.

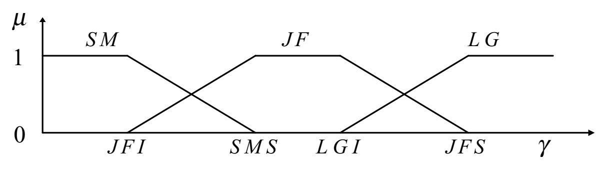


**Figure 6:** Membership function of antecedent.

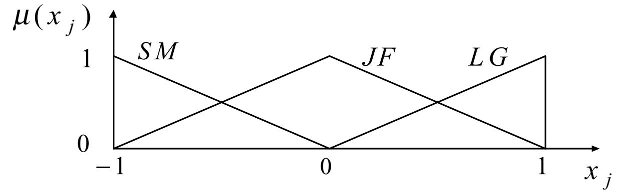


**Figure 7:** Membership function of consequent.

The fuzzy variable $\gamma$ used in the antecedent of the fuzzy rule has a trapezoidal membership function, as shown in Figure 6. In addition, the membership function used in the consequent of the fuzzy rule has a triangular form, as shown in Figure 7. The horizontal scale in the membership function of Figure 6 can be selected as

$$JFI = \min(\gamma_f \cup \gamma_s)\ ,\quad SMS = \min(\gamma_f \cap \gamma_s)\ ,$$

$$LGI = \max(\gamma_f \cap \gamma_s)\ ,\quad JFS = \max(\gamma_f \cup \gamma_s)\ . \qquad (51)$$

The complete inference is found by calculating the gravity of (36) in terms of membership function shown in Figure 7.

The transient characteristic is improved by using the tracking model with the binomial coefficient as follows:

$$A_m(q^{-1}) = (1 - \alpha\, q^{-1})^3\ ,\quad B_m(q^{-1}) = A_m(1)\ , \qquad (52)$$

where $\alpha$ is the adjustable parameter of $\alpha < 1$. The relation of settling time and stable adjustable pole is obtained through the step response simulation of the following tracking specification:

$$G_{tr} = \frac{q^{-2} A_m(1)}{A_m(q^{-1})} \cdot \frac{B(q^{-1})}{B(1)}\ . \qquad (53)$$

In this example, it is evident through the previous simulation that the tracking characteristics of both plants $\Sigma_f$ and $\Sigma_s$ have similar settling time, even if the plant parameter changes greatly. By examining each relation between the performance weight, the settling time and the tracking model, the transient characteristic is improved by tuning the pole $\alpha$ of the tracking model by the following formula:

$$\alpha(k) = a \log R(k) + b\ ,\quad a = 0.1350\ ,\quad b = -1.437 \times 10^{-2}\ . \qquad (54)$$

## 4 Simulation Results

The simulation result of the conventional adaptive pole placement control having the closed-loop pole designed by an optimal servo is shown in this section. After that the proposed auto-tuning result of adaptive pole placement control system is shown, even if the unstable compensator appears on the conventional system in the case that the plant parameter changes greatly. Finally, the transient response is improved by tuning the tracking model according to the weight in optimal servo design. It is assumed that the prefilter of (8) is specified and the reference signal of $r_m(k) = u_m(k)$ without reference model is used for the plant of (40). The weights of performance index of (25) are chosen to be constant values of $Q = diag\,(100,100,100)$ and $R = 10^3$.

The plant parameter has been changed to $\Sigma_s(\varsigma = 0.4,\ \omega_n = 20)$ from $\Sigma_f(\varsigma = 0.1, \omega_n = 60)$ at a step of 400 and a minimal disturbance noise of variance $\sigma_n^{\,2} = 5\times10^{-6}$ has been added to the simulation. The result of the adaptive pole placement control by the conventional method is first shown in Figures 8 through 10.

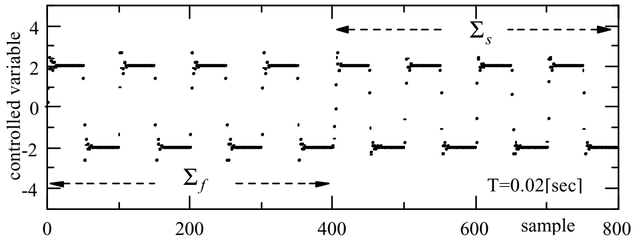


**Figure 8:** Controlled variable (conventional adaptive control).

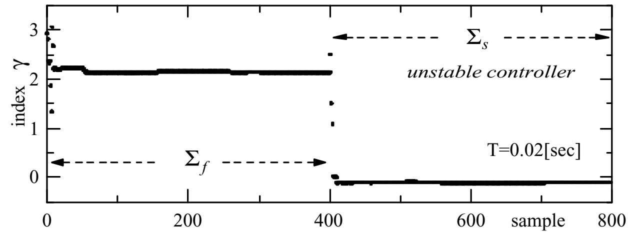


**Figure 9:** Stability index $\gamma$ (conventional adaptive control).

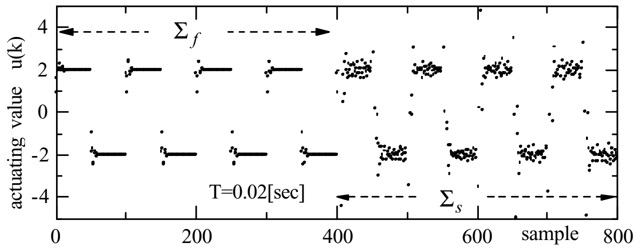


**Figure 10:** Actuating value (conventional adaptive control).

In Figure 9 the index $\gamma$ is very small in the area of the parameter $\Sigma_s$ and also the compensator is unstable. Therefore, the manipulated variable shown in Figure 10 is greatly disturbed even for a minimal random disturbance. Consequently, a strongly stable system is not achieved using the conventional method even if optimal servo system is designed to the given plant.

The simulation results obtained in the same condition mentioned above using the proposed method are next shown in Figures 11 through 13. The performance weight $R$ is adjusted appropriately and the appearance of unstable

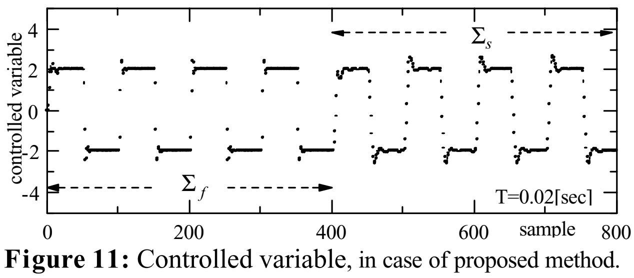


**Figure 11:** Controlled variable, in case of proposed method.

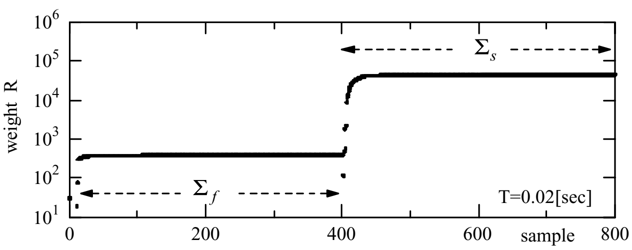


**Figure 12:** Weight R, in case of proposed method.

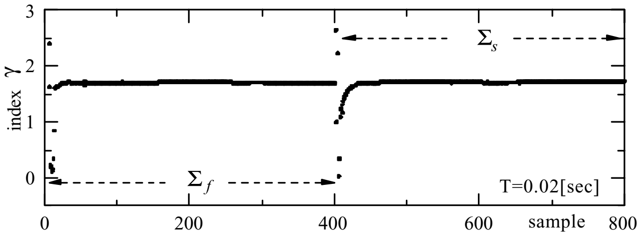


**Figure 13:** Stability index $\gamma$ , in case of proposed method.

controller can be prevented even after the plant parameters are changed greatly at a step of 400 in Figure 12. Furthermore, Figure 13 reveals that the proposed adaptive control system is appropriately adjusted by means of fuzzy inference as the stability index $\gamma$ is put in the specified region. An overshoot is observed in the transient response of a controlled variable in Figure 11 a little.

When the prefilter of (9) is specified and the tracking model of (4) is used, the transient response is finally improved as shown in Figures 14 through 16. The control variable is in good settling condition as shown in Figure 14 and also the overshoot disappear after the completion of parameter identification. Table 1 shows the difference of its characteristics when the reference model is employed. The settling time in both simulations is nearly the same. Simultaneously there is no disturbance in the actuating value as shown in Figure 15. Furthermore, the stable pole $\alpha$ of the tracking model is appropriately tuned as shown in Figure 16. The performance weight and the index $\gamma$ are almost similar to Figures 12 and 13.

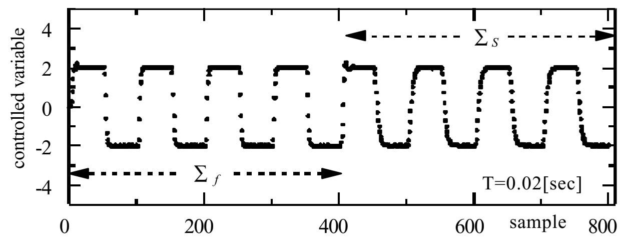


**Figure 14:** Controlled variable with tracking model.

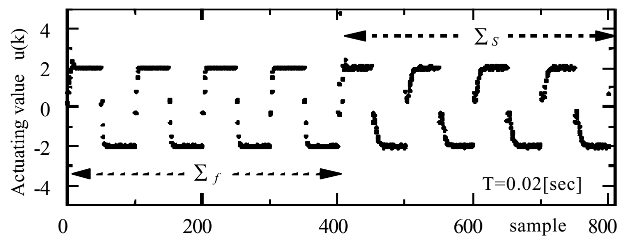


**Figure 15:** Actuating value with tracking model.

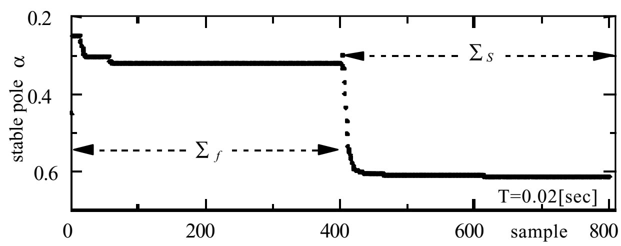


**Figure 16:** Stable pole $\alpha$ of tracking model.

**Table 1:** Characteristics of controlled variable.

| Evaluation / Simulation | | Overshoot $M_P$[%] | Peak time $t_P$[*sample*] | Settling time $t_S$[*sample*] |
|---|---|---|---|---|
| Result without Tracking model | $\Sigma_f$ | 14.8 | 4 | 8 |
| | $\Sigma_S$ | 24.8 | 10 | 21 |
| Result with Tracking model | $\Sigma_f$ | 0 | - | 9 |
| | $\Sigma_S$ | 0 | - | 20 |

## 5 Conclusions

The useful design method for the auto-tuning has been introduced to achieve a strongly stable system for the adaptive pole placement control. The difficulty of conventional adaptive pole placement control is how to place the stable pole to the *Diophantine* equation to design the appropriate controller according to the plant parameter that widely changes. The proposed methods can place the stability index in the specified area and then, overcome the problem of unstable series compensator that appears in conventional adaptive control system. The evident solution for the control purpose is easily achieved by applying the proposed scheme. Both operations of the estimation and the control were recursively repeated in order to inspect the real time performance; however, it does not necessarily require that they are simultaneously performed on every control interval in the auto-tuning of adaptive control system. In other words, the proposed method has been able to confirm even the sufficient applicability to practical machine.

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