Stabilization of a Wheel-Type Inverted Pendulum use Optimal Regulator

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1. Introduction

A pendulum is an apparatus consisting of an object mounted so that it swings freely under the influence of gravity (Figure 1A). This is generally seen in pendulum watches (Figure 1A). On the contrary, an inverted pendulum is a pendulum that has its center of mass above its pivot point (Figure 1B). However, because the center of mass is above the pivot point, an Inverted pendulum is an unstable system compare to a simple pendulum. Still, if forces are properly applied, using a linearity system of equations, the pendulum can be kept inverted and stable, and can therefore be used to check the control system of variable equipment. For example, Inverted Pendulum system is used in the *Segway* PT, which is a two-wheeled, self-balancing, battery-powered electric vehicle (Figure 1B) invented by *Dean Kamen* in 2001. But the weak point of the *Segway* is that it cannot keep stable without human control therefore causing some accidents. From this point we thought that it is important to apply a regulator theory to achieve a safer system for the *Segway*. Accordingly, this project is designed with the aim of investigating the effect of variation in weight and inertia in the make-up of a safer system. In this study, the Wheel-type Inverted Pendulum (WIP) is used as the control object and a control theory based on optimal regulator is applied.

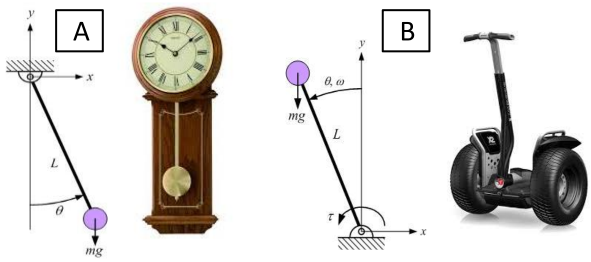


Fig 1. A- Simplified schematic of a pendulum and an example of pendulum watch. B- A schematic drawing of the inverted pendulum and a Segway PT.

2. Materials and method

In this project, we used an existing Wheel-type Inverted Pendulum body (Fig.2) constructed using aluminum columns that were cut and screwed; wheels and axle; a DC motor, and two encoders to read the angular variation for pendulum and an angular velocity of the wheels. Given that Wheel-type Inverted Pendulum (WIP) is unstable and can move only forward and backwards to improve the stability of the system, we used a step-adding weight controlled experiment. This consisted at adding subsequently masses of 0, 3, and 5 kg, and at each step, we determined the distribution of forces and derived an equation of equilibrium. Thus, we designed a model, summarized in Fig.3, from which we derived the non-linear equation necessary for the stability. By applying the *Euler-Lagrange equation* (Eq.1) to each mechanical energy of the system (WIP + added mass), we used (Eq.2), which was further derived to obtain the State linearization equation of motion (Eq. 3).

$$L = \frac{1}{2}\left(J_1 + M_1 r^2 + M_2 r^2\right)\dot{\alpha}^2 + \frac{1}{2}\left(J_2 + M_2 \ell^2\right)\dot{\theta}^2 + M_2 r \ell \dot{\alpha}\dot{\theta}\cos\theta - M_1 g r - M_2 g \ell \cos\theta - M_2 g r \quad (1)$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \alpha & \theta & \dot{\alpha} & \dot{\theta} \end{bmatrix}^T \quad (2)$$

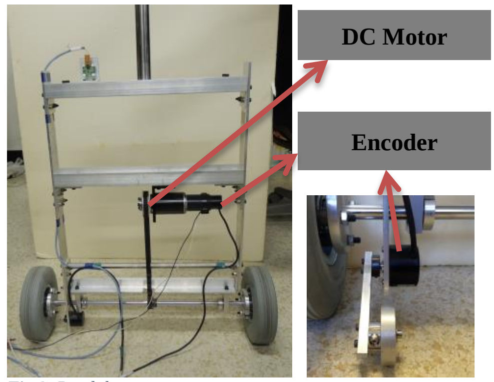


Fig 2. Pendulum system

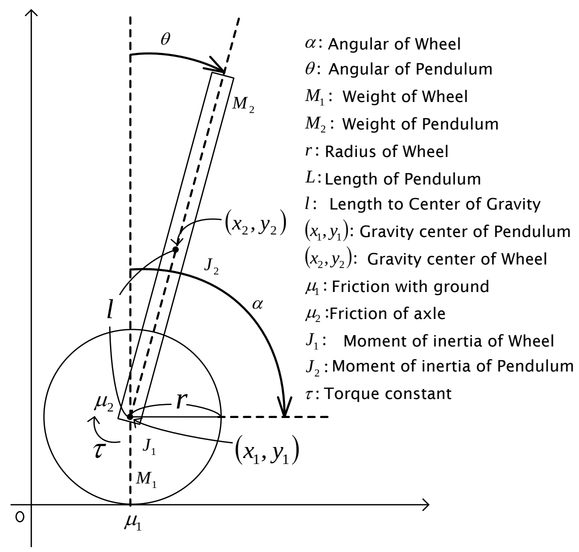


Fig 3. Schematics of our model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{11} & A_{12} & A_{13} \\ 0 & A_{21} & A_{22} & A_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ B_1 \\ B_2 \end{bmatrix} \tau \quad (3)$$

$A_{11} = -(M_2 r l)(M_2 g l)/\Delta$ $\quad A_{12} = -(J_2 + M_2 l^2)\mu_2/\Delta$

$A_{13} = (M_2 r l)\mu_1/\Delta$

$A_{21} = (J_1 + M_1 r^2 + M_2 r^2)(M_2 g l)/\Delta$

$A_{22} = (M_2 r l)\mu_1/\Delta$ $\quad A_{23} = -(J_1 + M_1 r^2 + M_2 r^2)\mu_2/\Delta$

$B_1 = (J_2 + M_2 l^2)/\Delta$ $\quad B_2 = -M_2 r l/\Delta$

$\Delta = (J_1 + M_1 r^2 + M_2 r^2)(J_2 + M_2 l^2) - (M_2 r l)^2$

3. Control System Design

We composed an Optimal Regulator to determine the parameters of (Eq.3). Fig.4 is a Block Diagram of the Optimal Regulator. At this point, (Eq.3) can be summarized as a state equation into (Eq.4), in which *u* represents input power as a function of the feedback gained (F) in (Eq.5). To determine the A in (Eq.4), we obtain a control input, such as to minimize the weighting matrix Q and R of (Eq.6).

$$\dot{x} = Ax + Bu \quad (4) \qquad u = -Fx \quad (5)$$

$$J = \int_0^{\infty} (x^T Q x + u^T R u) \qquad (6)$$

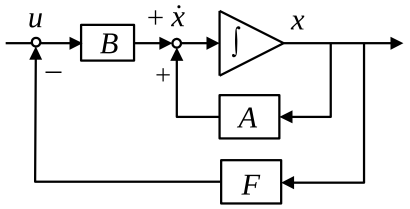


Fig.4 Block Diagram of Optimal Regulator

4. Controller Design

The controller design usually used in this type of study is the *Matlab* application. So we designed a *Matlab* program based on (Eq.3) and values of Table.1 that we used to derive the Feedback gain *F*. Logically, if this feedback gain is deriving by changing the Q and R values in (Eq.7), we can obtain the optimal feedback ***F*** values as shown in Equation 8. However, *Shimoyama* et al., (2011) indicated that it is difficult and even impossible to directly derive a feedback gain. So, they proposed that this can be possible by trial and error repeated experiments. Following this method, we could obtain the results shown in (Eq.8).

$$Q = diag[200 \quad 50 \quad 1 \quad 10], R = 3000 \qquad (7)$$

$$F = [-0.25819 \quad -14.529 \quad -0.38629 \quad -2.7313] \qquad (8)$$

Table.1 Parameter of used in the computation

M_1	Mass of Wheels	2.597[[kg]
M_2	Mass of Pendulum	4.386[[kg]
r	Radius of Wheel	0.089[m]
L	Length of Pendulum	0.55[m]
l	Position of the center of Gravity	0.33[m]
μ_1	Friction with the ground	0.354[N]
μ_2	Friction of axle	0.0001[N]
J_1	Moment of inertia of the Wheel	J_1=2(M_1 r^2/2)[kgm^2]
J_2	Moment of inertia of the Pendulum	J_2=(M_2 L^2)/12[kgm^2]
τ	Torque constant	2.88[Nm]
g	Gravity	9.8[m/s^2]

5. Primary delay differential filtering

In this project, Wheel angular and pendulum angular are getting by optical encoder while the angular velocity is obtained by the differentiation of the angular. However, this would create noise. So, we used the first-older lag differential filter of formula (9) to correct the noise. In addition, time constant of filter was used in the experiments δ=0.055. This value was obtained by the trial and error repeated experiments started from δ=0.055 until δ=0.01 as a result of removing the vibration and gradually raise the constant. *Shimoyama* et al., (2011) indicated that by simulation got it.

$$G_f(s) = \frac{s}{1+\delta s} \qquad (9)$$

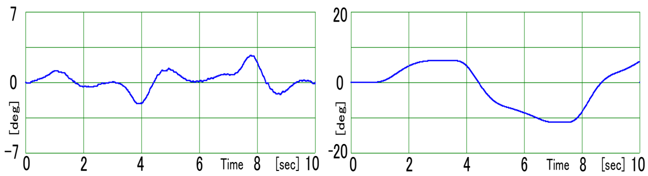
However, in the above formulas of this state, it cannot be used in the program. So, and we are subjected to bilinear transform. In addition, *T* of formula (10) is sampling interval. Formula (10) is angular velocity of wheels. Similarly, to derive angular velocity of pendulum, we can use these to get the data.

$$\dot{\alpha} = -\frac{T-2\delta}{T+2\delta}\dot{\alpha}(k-1) - \frac{2}{T+2\delta}\alpha(k-1) + \frac{2}{T+2\delta}\alpha(k) \qquad (10)$$

6. Experiment and results

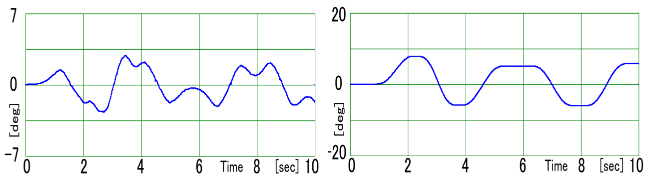
For the experiment, we set a sampling interval of T=0.001[sec] and a Time Constance $\delta_1 = 0.055$. After adding the extra weight, and we suggested the controller have to start on an angular of pendulum of 0.087 [rad]=5[deg]. We applied an optimal feedback of $F = [-0.25819 \quad -14.529 \quad -0.38629 \quad -2.7313]$ for all trial with 0 kg, 3 kg and 5 kg added.

After that, we tilt the pendulum started to use the optimal feedback F. The experiment result shown in Fig.5, Fig.6 and Fig.7. Fig.5 takes the position stability quickly to be inverted and Fig.6 takes more time but, Fig.7 with the passage time WIP will lose the control of stability. From this result, we observed that the stability of degradation is caused by changing weight and change of inertia.



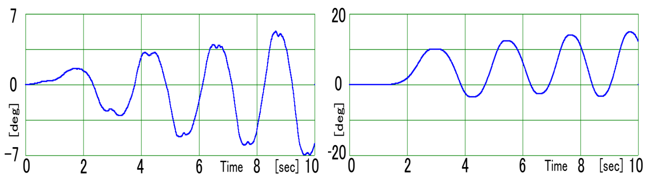
(a) Angular of pendulum (b) Angular of wheel

Fig.5 without the weight



(a) Angular of pendulum (b) Angular of wheel

Fig.6 With the weight of 3kg



(a) Angular of pendulum (b) Angular of Wheel

Fig.7 With the weight of 6kg

7. Conclusion

In this project, we focused on the influence on the stability by weight and change of inertia and experiments conducted. As a result, deterioration due to their stability could be demonstrated, because of a mismatch of design parameters. Also, in this project is designed with the aim of investigating the effect of variation in weight and inertia in the make-up of a safer system.

From those results, we need to try to keep an inverted position at a certain angle and we think to apply WIP on the adaptive control system.

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